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Trading Higher Software Piracy for Higher Profits: The Case of Phantom Piracy

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Faced with the sustained problem of piracy that costs nearly $40 billion in annual revenue losses, the software industry has adopted a number of technical, legal, and economic strategies to curb piracy and stem the resulting losses. Our work complements and contributes to the existing literature by exploring the possible effect of another economic lever—product bundling—on the relationship governing piracy and seller profits. The traditional economic rationale of demand pooling from bundling that enables sellers to extract higher surplus and its particular attractiveness for information goods with negligible marginal and bundling costs carry over to our analysis. However, the presence of piracy injects several new facets to our analysis. Bundling creates a shared level of piracy of disparate products, and under certain conditions to the detriment of one of the products. We argue that by construction of the copyright laws, the act of bundling itself can have a deterrence effect. This deterrence effect, along with shared piracy of products and demand pooling are ingredients that together dictate the overall piracy, pricing, profit, and welfare outcomes. Our analysis reveals several interesting insights. Bundling can be profitable even when the very act of bundling increases the piracy level of one of the products in the bundle. Termed *phantom piracy*, this represents a situation where sellers trade off higher piracy for one product in favor of lower piracy for the other product while deriving overall higher profits. Extensive simulation analysis shows that the region of phantom piracy is vastly expanded when additional products are introduced to the bundle. Conversely, under certain conditions, a profit maximizing seller opts not to bundle even when bundling can serve to lower the overall level of piracy. Price discounts that are typically offered by bundling are sharply deepened when piracy enters the equation. When piracy is a phenomenon to contend with, product bundling always increases consumer surplus even in scenarios where the seller may not realize higher profits. Unlike other forms of price discrimination that are often viewed by consumers with a jaundiced eye as they attempt to extract additional surplus from the consumers, product bundling in the software context can be a win-win scenario for both the buyers and the sellers.

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1. Introduction

Piracy or unauthorized reproduction has a significant deleterious impact on the digital goods industries including software, movies, music, and books. Although the technological innovations in the Internet and file-sharing technologies have recently opened the floodgates to facilitate the piracy of music and movies, the software industry has perennially borne the brunt of the problem from consumer piracy. Because the device on which packaged software products operate—a personal computer—is also the device that allows consumers to illegally reproduce software, the packaged software industry has been plagued by the problem of piracy since its inception nearly three decades ago. The ensuing revenue losses, estimated at $51.4 billion annually, are significantly larger than in any other digital goods industry.[1] Although the industry's vigorous efforts to combat piracy have been modestly successful—the global piracy rates have indeed dropped from about 50% in the late 1980s to 35% by the end of the 1990s—the piracy rates have stubbornly remained at this level since then with one in three software in use being illegal. At the same time, the estimated revenue losses continue to climb because of the overall growth of the software industry. With the piracy rates remaining steady at 35%, the financial losses have grown from about $30 billion in the year 2003 to about $40 billion in the year 2006.

[1] Business Software Alliance. Seventh annual BSA/IDC global software piracy study, http://portal.bsa.org/globalpiracy2009/studies/globalpiracystudy2009.pdf (last accessed August 30, 2010).

By many accounts software piracy remains the single largest concern faced by the software industry.

1.1. Modes of Piracy

The illegal act of copying digital goods without explicit permission from and compensation to the copyright holder is the underlying thread that governs the piracy of digital goods; however, the act of piracy itself can take many modalities. The modes of piracy can be categorized as *end-user piracy*, *Internet piracy*, and *counterfeiting*. The end-user piracy (alternately termed casual copying or license abuse) pertains to making unauthorized copies from a legal version above and beyond that permitted by the licensing agreement. In an individual use context, it involves an individual making unauthorized copies from a single license for family members, friends, or colleagues. The license abuse in a corporate setting stems from usage that is not concomitant and is beyond the usage levels permitted in the licensing contract. The Internet piracy entails obtaining illegal copies from Internet-based sources such as P2P sites, bulletin boards, or online auctions. This mode of piracy contrasts with the end-user piracy in that the entity providing the illegal copy has no explicit relationship and is often unknown to the individual obtaining the pirated copy. Finally, counterfeiting pertains to for-profit piracy wherein the illegal reproduction of the copyrighted material is conducted for the purpose of reselling the product in competition with the original copyright owner. This can take the form of hard disk loading where unauthorized copies of software are loaded onto computer hardware by computer sellers to make the purchase of the hardware more attractive. Another popular form of counterfeiting occurs through online auction sites and the Business Software Alliance (BSA) estimates that a very high proportion of software sold on auctions are pirated. In contrast to the profit motive that underlies counterfeiting, end-user piracy is driven by the goal of reducing the price paid by participants in acquiring a digital good. This can either take the form of an explicit cost sharing among participants or through more implicit quid pro quo arrangements (Gopal and Sanders 1997).

The introduction of P2P and file-compression technologies (most popularly in the form of MP3 format for digital music files) has significantly accelerated illegal sharing and procurement of digital music and videos (Bhattacharjee et al. 2006a, b; Gopal et al. 2006). Although the mode of Internet piracy has recently become the bane of the entertainment industry, the lion's share of losses for the software industry continues to emanate from end-user piracy.[2] This is because of the inherent complexities involved in the installation and usage of software products (Peitz and Waelbroeck 2006). The true cost of a software product is not only its purchase price but is also the cost incurred in ensuring that software installation does not lead to an unstable computing environment, the cost to learn to use the software to its full potential, and in making it work with other software. Often referred to as the total cost of ownership, this aspect of software makes the option of obtaining software through unknown and potentially unreliable sources on the Internet less appealing. As a result, end-user piracy, where the illegal software is obtained from known and reliable sources (i.e., where adequate technical and usage know-how is available and shared), constitutes the most prevalent mode of piracy of software products.

1.2. Antipiracy Measures

Expectedly, the software industry, along with other industries that are impacted by the piracy of digital goods, have taken aggressive steps to combat piracy and stem the resultant losses from unauthorized reproduction and use of their intellectual property. Drawing from a rich body of work in criminology, literature classifies the measures to combat piracy into *preventive*, *deterrent*, and *economic* strategies (Blumstein et al. 1978). These broadly map into *technical*, *legal/moral*, and *product and pricing* measures, respectively.

The objective of preventive measures is to use technical means to increase the costs of engaging in criminal activity. The examples of preventive control include hardware-based controls such as nonstandard disks, coder cards and hardware locks, and software-based controls such as special password codes and encryption (Malhotra 1994). Faced with the problem of illegal file sharing of music products, the music industry deployed the so-called *digital rights management* (DRM) in an attempt to control or prevent access to or copying of digital media, which can otherwise be copied with very little cost or effort. DRM employs encryption and authentication techniques to control access to and prevent unauthorized use of copyrighted content. In the software industry, the latest incarnation of preventive controls comes in the form of product activation.[3] It requires the user to communicate with the software producer to activate a product after installation and when copies of the software are installed on different machines. Consumers and various consumer advocacy groups have vehemently opposed these measures because they restrict the rights of usage and inconvenience legitimate users. The opponents argue that these techniques to combat

[2] http://www.bsa.org/usa/antipiracy/Product-Activation.cfm, http://www.siia.net/index.php?option=com_content&view=article&id=77&Itemid=7 (last accessed August 30, 2010).

[3] http://www.microsoft.com/piracy/activation.mspx.

illegal behavior may themselves be illegal as they violate "fair use" and other legal rights of consumers, and because of the "presumption of guilt without fair trial" that these measures imply. In addition, these measures are not foolproof and can be overcome by determined pirates. For example, hackers could disassemble the code and bypass the licensing calls completely, as was the case with Ocean's *Robocop* 3 game software for the Amiga platform (Myles and Nusser 2006). The techniques to defeat the preventive controls can easily be transferred through vehicles such as the Internet (Antonoff 1987). Further evidence against preventive controls comes from recent research that suggests that even when these measures are effective in curbing piracy, they do not necessarily translate into increased sales of the legal product (Gopal and Sanders 1997). Historical evidence suggests that these measures have not proven to be a consistently useful tool in the arsenal to combat piracy: with "tried but failed" being the modal outcome, including the most recent case of DRM in the music industry.

Deterrent measures draw from the legal protection offered to the creators of intellectual property from the copyright laws and the related laws in the Title 17 of the United States Code that aims to reward and foster innovation. Deterrent controls encompass educational, investigative, and legal campaigns to enforce the law and dissuade users from engaging in the act of piracy. The strategy of deterrence aims to increase both the likelihood of detection of illegal activity and the severity of punishment when caught. The digital goods industries have aggressively lobbied the U.S. Congress to strengthen the copyright laws and increase the penalties for violation. In 1992, the U.S. Congress passed the Software Copy Protection Bill that elevated software piracy from a misdemeanor to a felony. Penalties for felony convictions include fines of up to $250,000 and imprisonment for up to five years. In addition to the actual damages, the copyright owner is entitled to *statutory damages amounting to* $150,000 *per program copied*. The BSA and Software & Information Industry Association, representing over 1,200 software companies, actively pursue investigative campaigns by setting up piracy hotlines, filing lawsuits, and engaging in educational campaigns by disseminating information on the legal aspects of software piracy. For example, the industry provides training and audit tools for corporate customers, such as KeyAuditor,[4] to keep track of unlicensed software or to detect violations of any software licenses. The BSA estimates that the drop in software piracy rates are in large part attributable to better awareness, better defined copyright laws, and more specific and severe punishments for perpetrators (Schonwald 2003). In the music industry, although the use of DRM technologies appears to be on the decline, the legal actions against file sharers and the facilitators of file sharing (such as P2P network operators) continues unabated.

Economic measures begin with the underlying economic drivers of piracy to fashion pricing and product strategies that lower piracy and increase seller profits. Economic models of piracy, in general, study the impact of piracy on profits and, in particular, the effect of enforcing copyright. A growing body of empirical evidence suggests that the price of software is a leading determinant of piracy (Im and Van Epps 1991, Cheng et al. 1997, Chiang and Assane 2002, Hinduja 2003, Marshall 2007). The software industry has practiced both second-degree price discrimination in the form of site licenses and volume discounts, and third-degree price discrimination by pricing according to customer segments (e.g., students). Surprisingly, although this is commonplace in the United States, the software industry has traditionally followed a *globally uniform pricing* policy where a piece of software costs the same regardless of the country where it is sold. Based on a model that incorporates income effects, Gopal and Sanders (2000) show that this uniform pricing policy can in large part explain the wide disparity in piracy rates across countries.

An important strand of research suggests that piracy is not necessarily always harmful to a seller. In the presence of strong network effects, where a consumer's willingness to pay increases with the total installed base (which includes both legal and pirated versions), a software firm can gain by allowing or even promoting piracy. The network effects in software arise from the need for software to communicate and work with other software, and from the need for users to communicate and interact with other users. Network effects can also arise from word of mouth where a pirate knowledgeable about the software can help spread the knowledge, reduce the cost of adoption, and encourage others to purchase the software (Conner and Rummelt 1991, Takeyama 1994, Belleflamme 2003, Gayer and Shy 2003). However, some argue that network effects, although strong in the past, have been significantly reduced because of the standardization of file formats (Peitz and Waelbroeck 2006). Interestingly, Sundararajan (2004) suggests that even in the absence of network effects, the *profit maximizing* level of technological protection is strictly lower than the *technologically maximal* level of protection when technological protection lowers the value of a legal product.

1.3. Focus of Our Work

Our work complements and contributes to the existing literature by exploring the possible effect

[4] http://www.sassafras.com/keyaudit.html.

of another economic lever—product bundling—on the relationship governing piracy and seller profits. Because software is an information good with negligible marginal and bundling costs (Bakos and Brynjolfsson 1999, 2000), bundling can be an attractive option for the seller to pool demand and extract higher surplus. In addition to the traditional effects of bundling, the presence of piracy introduces several new effects. The act of bundling creates a shared level of piracy of disparate products, and under certain conditions can increase the piracy levels of one of the products in the bundle. By construction of the copyright laws and by the letter of the law, the act of bundling itself can serve as an added deterrent. This deterrence effect, along with shared piracy of products and demand pooling, determine the overall piracy, pricing, profit, and welfare outcomes. Our analysis reveals that bundling can be profitable even when the very act of bundling increases the piracy level of one of the products in the bundle. Termed *phantom piracy*, this represents a situation where sellers trade off higher piracy for one product in favor of lower piracy for the other product, while deriving overall higher profits. Extensive simulation analysis shows that the region of phantom piracy is vastly expanded when additional products are introduced to the bundle. Price discounts that are typically offered by bundling are sharply deepened when piracy enters the equation. When piracy is a phenomenon to contend with, product bundling always increases consumer surplus even in scenarios where the seller may not realize higher profits. This finding adds a new wrinkle to debate on antitrust scrutiny of bundling in the software industry, made famous by the legal troubles faced by Microsoft (Gilbert and Katz 2001). Whereas bundling can be construed as anticompetitive behavior, the threat of piracy lends to a more benign and practical rationale for bundling. Bundling can be an effective economic tool to combat piracy, and when effective it leads to lower prices, which then percolate to increased consumer and social surplus.

The remainder of this paper is organized as follows. Section 2 presents an economic model of consumer piracy and the ensuing seller profits with a single product. Section 3 presents the analysis of bundling of two products and develops the price, piracy, profit, and welfare implications of product bundling. Section 4 extends this to bundling additional products through simulation analysis. Section 5 presents concluding remarks and offers suggestions for further analysis.

2. Model of Consumer Piracy

From the economic vantage point, piracy is manifested from a user's endeavor to maximize the net value of acquiring the software. The option of procuring a software product at a price lower than the market price comes from the fact that the marginal cost of production is minimal and the consumption utility of the software does not diminish when it is shared with others. These characteristics, along with the observation that end-user piracy constitutes the archetypical mode in which piracy of software takes place, naturally lead to models of software *sharing clubs* (Gopal and Sanders 1997, 1998; Bakos et al. 1999; Varian 2000). With characteristics similar to a "private goods club" (Buchanan 1965), one legitimate copy of the software product is shared among the members of the club.

Let p_m denote the market price of a software product, p_i denote the component of price that each individual member of a club has to pay, b denote the number of products in a product bundle, c denote the strength of the deterrent controls employed by the software seller, and $g(c, b)$ denote the effective deterrent control, which is a nondecreasing function of bundle size b and deterrent control c. The effective price paid by an individual in a club of size n can be represented as

$$p_i = \frac{p_m}{n} + g(c, b)(n-1). \tag{1}$$

Before proceeding further with the model development, we describe several key characteristics of the above specification.

(i) Congestion—lowered utility to each member as additional members join—forces a typical private goods club to limit membership. The congestion in software sharing club results from the deterrent controls employed by the seller and the legal system under which the software is purchased and consumed.

(ii) The copyright laws draw no distinction between the individual that owns a legal product but willfully provides copies for others and the individual that obtains the pirated version. Both are equally culpable in the eyes of the law.

(iii) The cost to an individual due to deterrent controls—the threat of detection and penalty when legal action is pursued—increases monotonically with the deterrent controls employed by the seller, size of the club, and number of products in a product bundle. By inducing this negative externality, the model captures the fact that larger clubs and clubs sharing a large number of products are easier to detect and more economical for sellers to pursue legal action. Statutory laws permit the copyright owner damages of up to \$150,000 *per program copied*, in addition to the losses accrued from lost sales. Note that there is also a positive externality of larger club size, i.e., a drop in individual share of market price, which is reflected in the first term of Equation (1).

(iv) Given the sporadic use and the controversy surrounding preventive controls for software products, preventive costs are not included in the model. However, as modeled in Gopal and Sanders (1997), preventive controls impose additional costs to individuals, and inclusion of these in our model does not add or alter the qualitative insights derived from the current model setup.

Given the market price and deterrent controls, the software clubs will control their membership to balance the benefits of lowered price along with the threat of detection and ensuing legal sanctions. Thus, minimizing Equation (1) with respect to n yields

$$n^* = \sqrt{\frac{p_m}{g(c,b)}}, \tag{2}$$

where n^* is the optimal club size for a given p_m and $g(c,b)$. Note n^* indexes the "*extent of piracy*" with n^*-1 illegal copies of software in circulation for each legitimate copy of the software. Because $\partial n^*/\partial c < 0$, $\partial n^*/\partial b < 0$, and $\partial n^*/\partial p_m > 0$, it follows that the software publisher can reduce piracy through increased deterrent controls, increased number of products in a bundle, or by reducing the market price of the software. This approach to model the size of the software club is consistent with Asvanund et al. (2004), who develop a model for the optimal size of peer-to-peer file-sharing networks by incorporating both positive and negative network externalities.

Consider a single software product and let $d(p_i)$ denote the demand for the product, with $\partial d/\partial p_i < 0$. Let f denote the fixed costs in developing and marketing the software. With negligible marginal costs of production, the seller profits are expressed as

$$\Pi = \frac{p_m}{n^*} d(p_i) - f. \tag{3}$$

By setting $b=1$ in Equations (1) and (2), the above expression can be employed to derive profit maximizing price, for a given value of c. As shown in Gopal and Sanders (1997), the seller revenue R $(=\Pi + f)$ is strictly monotonically increasing in c when $\partial d/\partial p_i < 0$.

To provide specific functional forms, consider the case where the product valuations in the market are distributed uniformly $U(0, N/\alpha)$, as depicted in Figure 1. This results in a linear demand function as shown below:

$$d = N - \alpha p_i, \tag{4}$$

where N represents the maximum number of individuals interested in the product (i.e., zero-price demand), and the parameter α indexes the value of the product (the average and maximum valuation and the total demand for a given price are inversely related to α, as illustrated in Figure 1). With this functional specification of demand, the optimal piracy levels and seller profits are

Figure 1 **Normalized Uniform Demand Function**

$$n^* = \frac{N + \alpha g(c,1)}{4\alpha g(c,1)}, \tag{5}$$

$$p_m^* = \left(\frac{N + \alpha g(c,1)}{4\alpha g(c,1)}\right)^2 g(c,1), \tag{6}$$

$$\Pi^* = \frac{(N + \alpha g(c,1))^2}{8\alpha} - f. \tag{7}$$

In the analysis that follows we will continue to employ the specification of uniformity in product valuations across consumers. Although this is done primarily for analytical and expositional convenience, we have confirmed that the reported results continue to hold across various demand specifications, as long as the bundled offering serves to lower the demand variance. In addition, in the succeeding analysis we unceremoniously drop the fixed cost f from the seller profits. This is done in light of the fact that the incremental costs incurred in bundling digital goods are minimal to nonexistent, and the total fixed costs remain similar whether the products are offered individually or together in a bundle. Because the primary focus of analysis is a comparative analysis of bundling with disaggregated product offering, the fixed costs only add to the complexity of mathematical expressions without shedding any insights of significance.

3. Bundling and Piracy

The mutual impacts of piracy and bundling and their collective impact on seller profits are developed in this section. To provide a benchmark for performance comparison, we first develop the case where a multiproduct monopolist offers disaggregated products independently.

The seller offers two software products indexed with 1 and 2. Let α_1 and α_2 index the values of the two products being bundled. Also, without loss of generality, we assume $\alpha_1 > \alpha_2$. The piracy levels of the two products and the total seller profits can be computed as

$$n_1^* = \frac{N + \alpha_1 g(c,1)}{4\alpha_1 g(c,1)} \quad \text{and} \quad n_2^* = \frac{N + \alpha_2 g(c,1)}{4\alpha_2 g(c,1)}, \tag{8}$$

$$\Pi^*_{1,2} = \Pi^*_1 + \Pi^*_2 = [N^2(\alpha_1+\alpha_2) + 4N\alpha_1\alpha_2 g(c,1) + \alpha_1\alpha_2(\alpha_1+\alpha_2)g(c,1)]/(8\alpha_1\alpha_2). \quad (9)$$

The above functions lead to the following result.

Lemma 1. *If $\alpha_1 > \alpha_2$, then the profit maximizing piracy level for product 1, $n_1^* - 1$, is less than the profit maximizing piracy level for product 2, $n_2^* - 1$.*

When $\alpha_1 > \alpha_2$, the average and maximum valuation and the total demand for a given price are higher for product 2. Hence, product 2 can be viewed as higher valued than product 1. For convenience, we will index the valuation differences with the parameter β, where $\alpha_1 = \beta\alpha_2$. When $\beta = 1$, the products are identical in valuation with the relative value differences increasing with increasing values of β. The lemma suggests that the higher valued product is pirated more.

Alternately, the seller can choose to offer the two products in a bundle. The density function of resulting valuations, $f_d(p_i, \alpha_1, \alpha_2)$, for the bundle is the convolution of the two independent uniform density functions. This density function, graphically demonstrated in Figure 2, is given by

$$f_d(p_i,\alpha_1,\alpha_2) = \begin{cases} \dfrac{\alpha_1\alpha_2}{N^2}p_i, & p_i \le N/\alpha_1; \\ \dfrac{\alpha_2}{N}, & N/\alpha_1 < p_i \le N/\alpha_2; \\ \dfrac{\alpha_1+\alpha_2}{N} - \dfrac{\alpha_1\alpha_2}{N^2}p_i, & \\ \quad N/\alpha_2 < p_i \le (N/\alpha_1 + N/\alpha_2). \end{cases} \quad (10)$$

The demand, d_b, for the bundled product demand (represented in the shaded region in Figure 2) when an individual user incurs a cost of p_i is given by[5]

$$d_b = N\left(1 + \frac{\alpha_2}{2\alpha_1} - \frac{\alpha_2}{N}p_i\right). \quad (11)$$

With the relationship between p_i and p_b, the market price of the bundled software is governed by

$$p_i = \frac{p_b}{n_b} + g(c,b)(n_b - 1). \quad (12)$$

The pursuant seller profit maximizing piracy level, price, and profits can be derived to yield the following results:

$$n_b^* = \frac{N(2\alpha_1+\alpha_2) + 2\alpha_1\alpha_2 g(c,2)}{8\alpha_1\alpha_2 g(c,2)}, \quad (13)$$

$$p_b^* = \left(\frac{N(2\alpha_1+\alpha_2) + 2\alpha_1\alpha_2 g(c,2)}{8\alpha_1\alpha_2 g(c,2)}\right)^2 g(c,2), \quad (14)$$

$$\Pi_b^* = \frac{(N(2\alpha_1+\alpha_2) + 2\alpha_1\alpha_2 g(c,2))^2}{32\alpha_1^2\alpha_2}. \quad (15)$$

3.1. Piracy Implications of Bundling

A software sharing club of size n_b^* contains one legitimate copy of each of the two products, and $n_b^* - 1$ pirated versions of each of the two products. When the products are sold independently, $n_1^* - 1$ pirated copies of product 1 and $n_2^* - 1$ pirated copies of product 2 are created from each legitimate copy of the software. The following proposition establishes the comparative piracy levels when the products are introduced in a bundled format. All proofs are provided in the appendix.

Proposition 1. *The piracy level of the higher priced product will be lowered by bundling (i.e., $n_b^* < n_2^*$) if*

$$\frac{g(c,2)}{g(c,1)} > \frac{2\beta+1}{2\beta}$$

and the total piracy of the two products will be lowered by bundling (i.e., $n_1^* + n_2^* > 2n_b^*$) if

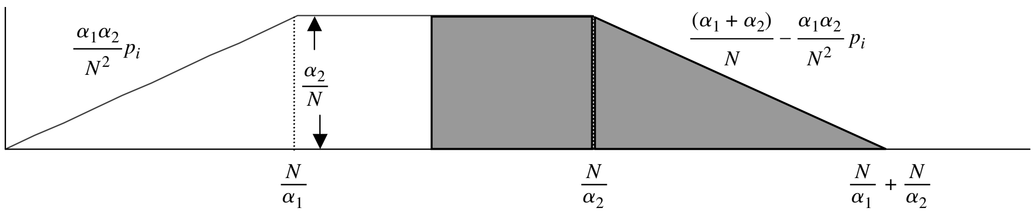
$$\frac{g(c,2)}{g(c,1)} > \frac{2\beta+1}{\beta+1}.$$

Proposition 1 immediately provides several key insights. First, bundling can lower the piracy level of the higher valued product as long as software producers have access to a higher level of deterrent controls for a bundle as compared to a single product. Second, bundling can also serve to reduce the overall level of piracy if the deterrent controls are higher for a bundle. Toward this end, it seems that the software piracy law that enforces penalties based on the number of pirated software is, from an economic perspective, an appropriate policy. Proposition 1 also indicates that when the difference in the value of two bundled products is large (i.e., β is large), although the piracy for larger valued products will be reduced by having marginally larger deterrent costs for the bundle, the overall piracy can be reduced if the deterrent cost for a bundle is twice that of a single product because $\lim_{\beta\to\infty}(2\beta+1)/(\beta+1) = 2$. Again, the current software laws that penalize equal amount for each violation should be able to reduce the overall piracy. Although Proposition 1 presents the conditions under which the level of piracy of the bundle will always be less than the piracy level of the higher valued item, it does not address the relative piracy level of the smaller valued item in the bundle. This is addressed in Proposition 2.

Proposition 2. *If $g(c,2)/g(c,1) > \beta + 1/2$, then $n_b^* < n_1^*$.*

[5] The demand function in Equation (11) is applicable only when $\beta \ge 3/2$. For mathematical exposition we impose this boundary condition on the value of β, although all of the subsequent analysis follows for all values of $\beta \ge 1$.

Figure 2 Demand for Bundled Product



Note that Proposition 2 implies (because $\beta \geq 3/2$) that $g(c,2)/g(c,1) > 2$, for the piracy level of the bundle to be smaller than the piracy level of the lower valued item in the bundle. The proposition also indicates that if the goal is to reduce the piracy level for both the products in a bundle, as the difference in valuation of products increases, the relative deterrent costs of the bundle need to increase. Together the two propositions suggest that when the products bundled are similar in valuation, bundling lowers the piracy level of each of the two products, as long as appropriate deterrence costs exist. When the valuation differences increase, at a given deterrence cost level, the seller in essence trades the lower rate of piracy for one product (product 2) for a higher rate of piracy for the other product (product 1) in an attempt to lower the overall levels of piracy. Whether such an outcome is desirable for the seller is addressed in the following subsection.

It is important to observe that the overall effect on piracy is a composite of price discounting and bundling. The bundling effect in this context refers to piracy implications when no price discount is offered with the bundle, i.e., when $p_b = p_1^* + p_2^*$. This pure act of bundling, without attendant price discount, can result in the lowering of the piracy levels. Let η_b denote the piracy level of the bundle when the price of the bundle is set at $p_1^* + p_2^*$. Applying Equation (2), we have

$$n_1^* = \sqrt{\frac{p_1^*}{g(c,1)}} \quad \text{and} \quad n_2^* = \sqrt{\frac{p_2^*}{g(c,1)}} \quad \text{and}$$

$$\eta_b = \sqrt{\frac{p_1^* + p_2^*}{g(c,2)}}.$$

From the above, we can easily demonstrate the following:

(i) when $g(c,2) > 4g(c,1)$, bundling (without price discounting) always lowers the overall piracy level of the bundle;

(ii) when $g(c,2) > 2g(c,1)$, bundling (without price discounting) always lowers the piracy level of the higher valued product;

(iii) when $g(c,2) > 2g(c,1)$, bundling (without price discounting) lowers the overall piracy level of the bundle when the relative valuation of products are not too far apart; and

(iv) when $g(c,2) > g(c,1)$, bundling (without price discounting) lowers the piracy level of the higher valued product when the relative valuation of products are not too far apart.

The total reduction in piracy of $n_1^* + n_2^* - 2n_b^*$ can be parsed into piracy reduction due to bundling (without price discounting), $n_1^* + n_2^* - 2\eta_b$, and piracy reduction due to price discounting, $2\eta_b - 2n_b^*$. The following proposition establishes the relative impacts of the two.

Proposition 3. *The piracy reduction due to bundling (without price discounting) increases with $g(c,2)/g(c,1)$ and the piracy reduction due to price discounting decreases with $g(c,2)/g(c,1)$.*

It can be verified that the piracy reduction due to both the effects increase as the difference in the relative valuation of the two products increases. The above results suggest that the effect of bundling on piracy (outside of the price effect) in essence arises from, and is strongly influenced by, the specification and the enforcement of copyright laws.

Note that $\eta_b > n_b^*$ and hence the overall piracy for the bundle is higher when no price discount is offered. However, this implies that each individual in fact pays a smaller fraction of the market price, because the bundled product is shared with a relatively larger number of individuals. An important insight here is the distinction between "price charged by the seller" and "price paid by a consumer." Even when the seller does not offer a price discount for the bundle, consumers alter their piracy behavior with the bundle to affect a net price discount. This is a fascinating insight because it reveals that piracy exhibits effects that are similar to the traditional "price effects" observed in the bundling literature.

3.2. Profit Implications of Bundling

Whether bundling is preferred by the seller is dependent on $\Pi_b^* - \Pi_{1,2}^*$. When this expression is positive, the seller chooses to bundle, otherwise the seller prefers offering two products independently. The following proposition establishes the key result on the profit implications of bundling.

Proposition 4. *The profits from bundling will be greater than those compared with selling the two products individually (i.e., $\Pi_b^* > \Pi_{1,2}^*$) when*

$$\beta < \frac{N^2 + 4\alpha_1^2 g^2(c,2) + 4N\alpha_1 g(c,2) - 4\alpha_1^2 g(c,1)}{4\alpha_1^2 g(c,1) + 8N\alpha_1(2g(c,1) - g(c,2))}.$$

It can easily be shown that the profit with bundling is larger when the piracy level of the bundle is below the piracy level of each of the two products in the bundle. Moreover, as $\Pi_b^* - \Pi_{1,2}^*$ is strictly decreasing in β, Proposition 4 provides a bound on the value of β such that as long as the relative valuations of the products are below this value the bundling will result in higher profits. However, bundling can be unprofitable ($\Pi_b^* - \Pi_{1,2}^* < 0$) even when $n_1^* + n_2^* > 2n_b^*$, i.e., when bundling reduces the overall level of piracy. For instance, as Proposition 1 shows, when $g(c,2)/g(c,1) \geq 2$, bundling always lowers the overall level of piracy even if widely disparate products are bundled together. However, Proposition 4 indicates that for large values of β the profit from bundling becomes lower than that from offering the two products independently. Juxtaposing the two results on piracy and profits naturally lead to the following zones of bundle-induced piracy:

Congruent Piracy: $\Pi_b^* - \Pi_{1,2}^* > 0$ and $n_b^* < n_1^*$. In this region, the piracy level of each of the two products is lowered and the profit is increased from bundling.

Phantom Piracy: $\Pi_b^* - \Pi_{1,2}^* > 0$ and $n_b^* > n_1^*$. In this region, piracy level of product 1 is increased and the seller profit is increased from bundling.

Incongruent Piracy: $\Pi_b^* - \Pi_{1,2}^* < 0$ and $n_1^* + n_2^* > 2n_b^*$. In this region, bundling lowers seller profits despite lowering the overall level of piracy from bundling.

Interestingly, the act of bundling may cause "lowering piracy" and "improving profits" to be not necessarily synchronous. The zone of congruent piracy does represent a win-win scenario where the piracy of both products is reduced and profits are consequently enhanced. When the valuations of the products result in phantom piracy, the seller disregards increased piracy of product 1 in an attempt to increase profits. When in the region of incongruent piracy, the seller refrains from bundling despite the fact that it may reduce the level of piracy of the higher valued product. Bundling in essence "couples" a lower valued product with a higher valued product and thus serves to increase the piracy cost and lower the piracy of the higher valued product. In the phantom piracy zone, the seller is better off with such bundling even when it increases the piracy levels of the lower valued product. However, there is a limit to when this type of bundling is beneficial. If the valuations of the two products are too disparate, bundling these products hurts the seller profits despite the fact that the total piracy is lowered because of bundling.

3.3. Price Implications of Bundling

A perusal of Microsoft pricing shows that the Office bundle retails at \$499 while the individual software Word, Excel, PowerPoint, and Access that comprise the bundle each retail at \$229 (as of early 2007). This pricing scheme translates to a hefty 45% discount for the bundled offer. Although it is well established that sellers offer a price discount when the products are sold as a bundle (bundle price is lower than the sum of the individual prices), we examine whether and to what extent piracy impacts the level of discount that is offered. Does piracy increase or decrease the level of discount that would otherwise be offered?

Proposition 1 indicates that the overall level of piracy can be controlled if $g(c,2) \approx 2g(c,1)$ regardless of the difference in valuations of the two products in the bundle. Incidentally, the U.S. software protection bill is also consistent with this result, i.e., the provision of *statutory damages amounting to* \$150,000 *per program copied*. Therefore, although the results on the parameter bounds presented in subsequent analysis can be derived for general values of $g(c,b)$, for parsimony and interpretability in rest of this paper, we assume that $g(c,1) = c$ and $g(c,b) = bc$ (i.e., $g(c,2) = 2c$).

To provide a baseline for this comparison, we first consider the scenario without the threat of piracy. In such a situation the individual price paid by a consumer p_i is identical to the market price charged by the seller. With no piracy threat, the seller profits with individual products are

$$\Pi_1 = p_1\left(1 - \frac{\alpha_1 p_1}{N}\right) \quad \text{and} \quad \Pi_2 = p_2\left(1 - \frac{\alpha_2 p_2}{N}\right), \qquad (16)$$

and with the bundled product,

$$\Pi_b = p_b\left(1 + \frac{\alpha_2}{2\alpha_1} - \frac{\alpha_2}{N} p_b\right). \qquad (17)$$

Upon obtaining the profit maximizing prices, the ratio of bundle to individual product prices can then be calculated as

$$\left(\frac{p_b^*}{p_1^* + p_2^*}\right)_{\text{no-piracy}} = \frac{2\beta + 1}{2\beta + 2}. \qquad (18)$$

Employing Equation (2) we can write

$$\left(\frac{p_b^*}{p_1^* + p_2^*}\right)_{\text{piracy}} = \frac{g(c,2)}{g(c,1)} \frac{(n_b^*)^2}{(n_1^*)^2 + (n_2^*)^2}. \qquad (19)$$

With the conditions that $n_b^* \geq 1$, $n_1^* \geq 1$, and $n_2^* \geq 1$.

Proposition 5. *If* $c < N/(3\alpha_1)$, *then*

$$\frac{\partial}{\partial c}\left[\left(\frac{p_b^*}{p_1^* + p_2^*}\right)_{\text{no-piracy}}\right] > 0.$$

Proposition 6. *If* $c < N/(3\alpha_1)$, *then*

$$\left(\frac{p_b^*}{p_1^* + p_2^*}\right)_{\text{no-piracy}} > \left(\frac{p_b^*}{p_1^* + p_2^*}\right)_{\text{piracy}}.$$

The condition $c < N/(3\alpha_1)$ implies that $n_1^* > 1$ and $n_2^* > 1$. Therefore, the above propositions describe the price implications of bundling two products that would be pirated when offered separately. Proposition 5 illustrates that as the extent of piracy increases (i.e., lower values of c), the seller offers larger discounts on the bundled product. Proposition 6 shows that the optimal bundle price of products susceptible to piracy is always lower than that for products that do not face the threat of piracy. Thus, the price discounts that are typically offered when bundling products are larger if the products are susceptible to piracy and this discount increases as the extent of piracy increases. Intuitively because bundling is an effective antipiracy instrument in that the overall piracy and the piracy level of at least one product is reduced, the seller who bundles products can consequently lower the price as the threat from piracy is lowered. This result is similar to the effect of legal deterrents in that the seller lowers the prices as deterrent costs increase.

3.4. Welfare Implications of Bundling

So far we have shown that the seller can leverage the piracy-dampening effect of bundling to both lower prices and increase profits. We now turn to the issue of welfare implications of bundling. Whereas the software producers' profits increase in the regions of congruent and phantom piracy, it is not clear what happens to consumer welfare.

For expositional purposes, we delegate the derivation of welfare results to the appendix and resort to simply stating the results in this discussion. When the products are sold independently, the total consumer welfare is given by

$$W_1 + W_2 = \frac{\alpha_1(p_{1i}+c)^2 + \alpha_2(p_{2i}+c)^2}{2}, \tag{20}$$

where W_1 and W_2 are the consumer welfare for each of the two products. When bundled, the consumer welfare is given by

$$W_b = \frac{N^2 - \alpha_2^2 p_{bi}^2}{2\alpha_2} + \frac{N^2}{\beta^2\alpha_2}\frac{4\beta^2+7\beta+2}{6} - \alpha_2 p_{bi}^2 - 2\alpha_2 c p_{bi}. \tag{21}$$

Proposition 7. $W_b \geq W_1 + W_2$.

The general concerns with the practice of price discrimination, that they increase producer surplus at the expense of consumer surplus, do not carry over to the case of bundling in the software context. In fact, Proposition 7 shows that consumer welfare is increased for any arbitrary differences in the valuations of the products in the bundle. As shown later in simulation analysis, this happens because of increased market coverage, i.e., because of increase in total number of units sold. Even when the products are beyond the value difference as specified by the bounds of phantom piracy, the consumers are better off with a bundle, though the seller may not be. However, bundling creates a win-win condition for both the producer as well as the consumers when the bundle is at least in the region of phantom piracy.

4. Beyond Two

In this section, we turn our attention to piracy and profit implications of bundles with three products. As the underlying analytical formulations for cases with bundles of more than two products become intractable, we conduct Monte Carlo simulations to derive results. Although we can study the outcomes for a variety of conditions, we chose to focus our attention at the intersection between phantom piracy and incongruent piracy. As one crosses from the phantom piracy zone to the incongruent piracy zone, bundling becomes unprofitable despite lowering the overall piracy levels. The questions we ask are as follows: Does adding a third product increase or decrease the region of phantom piracy? How does the valuation of the third product impact bundling and profits? How does mixed bundling impact piracy?

We begin the analysis with two products and set the parameter values such that the bound stated in Proposition 4 is realized. The values chosen are $\alpha_1 = 100$, $\alpha_2 = 2$, $c = 6.8357$, and $N = 1{,}000$. This case theoretically lies at the boundary of phantom and incongruent piracy regions (as defined in Proposition 4) where the seller is indifferent between bundling and offering two of these products individually.

We then simulate the addition of a third product whose valuation ranged from 1/75th of the higher valued product ($\alpha_3 = \alpha_2 \times 75 = 150$) to 40 times the value of the higher valued product $\alpha_3 = \alpha_2/40 = 0.05$. To avoid scaling problems, we present the results by breaking down our test cases into two sets:

Case 1. The third product added has a comparatively very high valuation ($0.05 \leq \alpha_3 \leq 1.5$). This represents the case where the third product is valued at equal to 40 times to about 1.33 times the higher valued product in the bundle of two products.

Case 2. The third product added has a low to medium valuation ($5 \leq \alpha_3 \leq 150$). This range covers the gamut where the third product has a valuation lower than the lower valued product in the bundle to higher than the lower valued product in the bundle but always has the lower value than the higher valued product in the bundle.

In the simulations, besides adding a third product, we also explore a mixed-bundling strategy as well because analytical treatment of mixed bundling becomes intractable in our modeling framework.

In the mixed-bundle strategy, the producer sells all possible bundles of products in the market including individual products, various two-product bundles, and a three-product bundle. For mixed bundling we ensure that there is no cannibalization of higher price bundles by lower price bundles, i.e., we set the price for lower price bundles in such a way so that customers who can buy high price bundles have higher surplus as compared to lower price bundles thereby satisfying consumers' incentive compatibility constraint.

Each simulation scenario, with a different value of the third product, is replicated 10 times in the simulation. The simulation explores four different scenarios:

(i) when all products are sold individually;

(ii) when there is a bundle of product 1 and 2, but product 3 is sold individually;

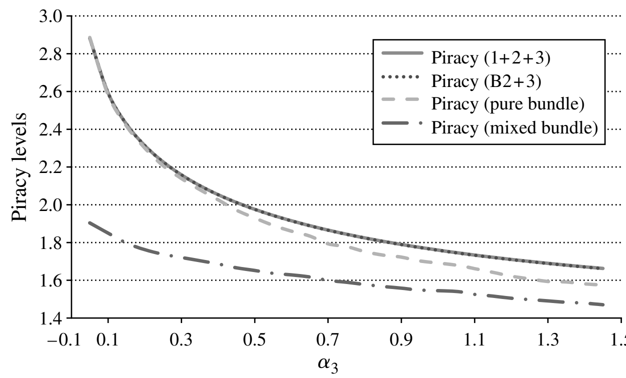
(iii) when all the three products are bundled;

(iv) when there is a mixed-bundling strategy, i.e., the products are sold individually, in all the three possible bundles of two products and a bundle of three products.

Overall, we ran the simulation for 50 values of the third product, resulting in a total of 2,000 simulations (four scenarios with 10 replications each) with valuations generated for a population size of 1,000. In each replication of the simulation, we draw a random valuation from a uniform distribution $U(0, 1{,}000/\alpha_i)$ for each product for each customer. The valuations are drawn from an independently seeded distribution to avoid any overlap of random number stream. Therefore, in each iteration, each of the 1,000 simulated customers have a certain valuation for each product. The optimal prices for the case are then determined by numerically maximizing the profit of the seller, accounting for the club size and deterrent control. For the purpose of the simulation, we chose $g(c, b) = bc$. Therefore, while the simulation model adheres to the effective price structure of Equation (1), the optimal prices and bundle sizes are numerically determined.

Figure 3 presents the piracy ramifications in Case 1 when the third product with extremely high valuation is added to the bundle. The figure shows the total piracy rates (on a logarithmic scale) for the four scenarios. The figure compares the average piracy levels per legitimate copy of the product. For the case of pure bundles, the average piracy is simply the club size. However, for the individual product sale case, this can be computed as the total number of users for all the three products individually divided by the total number of the products. For the mixed-bundling case, we use the intuition from the average piracy computation for individual product sale scenario and compute the average by computing the total users from sales of all bundles divided by the total number of legitimate copies sold. Formally, let B define

Figure 3 **Piracy Levels When the Third Product Has Very High Valuation**



the exhaustive set of all the product bundles available under mixed bundling, u_k be the number of users of a particular bundle $k \in B$, u_t be the total number of users of all the products, s_k be the legitimate products sold for bundle $k \in B$, and s_t be the total number of legitimate products sold. Note that $u_t = \sum_B |k| u_k$, where $|k|$ represents the size of the bundle k. Similarly, $s_t = \sum_B |k| s_k$. The average piracy level is defined as u_t/s_t.

It is clear from Figure 3 that piracy levels with mixed bundling are significantly smaller compared to the other cases regardless of the value of the third product, and the piracy levels of selling products individually or a bundle of products one and two with the third product being sold individually are indistinguishable from each other. This is a surprising result and somewhat counterintuitive because the piracy rates of, for example, at least one of the individual products are usually higher than bundle piracy rates even in phantom piracy regions. However, the main driver of this result is that in the presence of bundles (i) there is relatively much lower demand for individual products, and (ii) the optimal prices of subbundles (two-product bundles) and individual products under mixed bundling is much smaller than the optimal prices when the large bundle is absent. The combination of these two factors results in a smaller piracy levels overall. Figure 3 also reveals several interesting results regarding pure bundles. Although the piracy levels of pure bundles, i.e., case (iii), at extremely high values of product 3 are virtually indistinguishable from cases (i) and (ii), as the product 3's value comes closer to that of product 2, pure bundling results in lower piracy rates.

Figure 4 provides the corresponding relative profits of cases (ii), (iii), and (iv) as compared to the base case of selling the products individually. Figure 4 provides several interesting insights. First, the profits under case (ii) are not significantly higher than that of base case, although they are consistently higher. On the other hand, the profits with pure bundling are

Figure 4 Relative Bundle Profits When the Third Product Has Very High Valuation

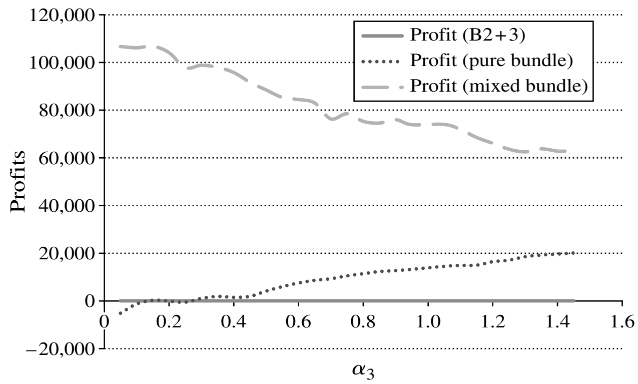
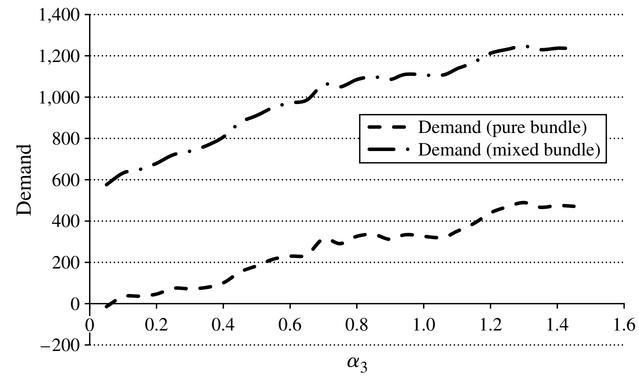


Figure 6 Relative Consumption When the Third Product Has Very High Valuation



smaller than the profits of selling the products individually for extremely high values of product 3. In other words, extreme values of products in a bundle do create the region of incongruent piracy (the difference between the low value product and the highest value product in this case is 2,000 times). The figure also indicates, as expected, that mixed bundling provides significantly higher profits than any other strategy; however, the difference of profits between the mixed bundling and pure bundling narrows as product 3 comes closer to the value of product 2 (the larger valued product among the other two products). In general, the simulations indicate that the general trend of increasing phantom piracy region continues by adding more products to a bundle.

In addition to piracy and profits, we also explore the issue of consumer surplus. As shown in Proposition 7, for two products the consumer surplus is greater with pure bundling. However, the consumer surplus implications when a third product is added are not clear. The results, illustrated in Figure 5, indicate that whereas the consumer surplus is virtually indistinguishable among various strategies when the value of product 3 is very high, the consumer surplus is higher with pure bundling and mixed bundling as products 2 and 3 come closer in valuations. Again, mixed bundling results in higher surplus when compared to pure bundling. The reason behind this result is the increase in market consumption; Figure 6 provides the total consumption of products 1, 2, and 3 with pure and mixed-bundling strategies as compared to selling the products individually. As the figure shows, the relative consumption increases with both bundling strategies (more with mixed bundling) as the product 3 comes closer to product 2 in valuation.

The objective of the second set of experiments was to assess whether congruent piracy where piracy levels of each of the products is lowered as a consequence of bundling is achievable in a bundle of three products. Case 2 is analyzed with particular focus on the regions where $\alpha_i/\alpha_j < 1.5\ \forall\, i \neq j;\ i, j = 1, 2, 3$.

Figure 7 illustrates the results of this analysis. The results indicate that the piracy level of the pure bundle and mixed bundles are significantly lower as compared to selling the products individually with mixed bundling resulting in lowest levels of piracy. As discussed earlier, although it is possible to have piracy levels with bundling being higher than that of nonbundled environments, the results indicate that a bundle's overall piracy level will be less than all

Figure 5 Consumer Surplus When the Third Product Has Very High Valuation

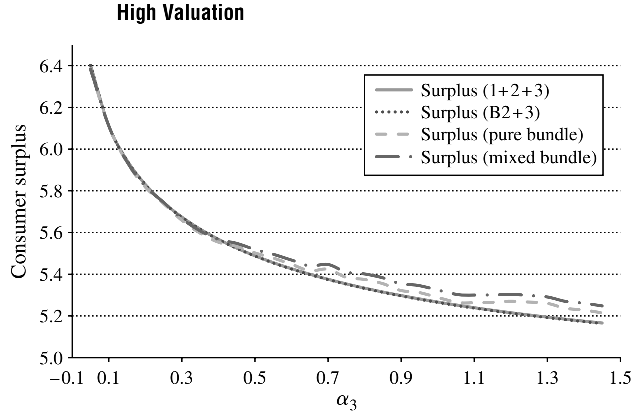


Figure 7 Piracy Levels When the Third Product Has Relatively Dispersed Valuation

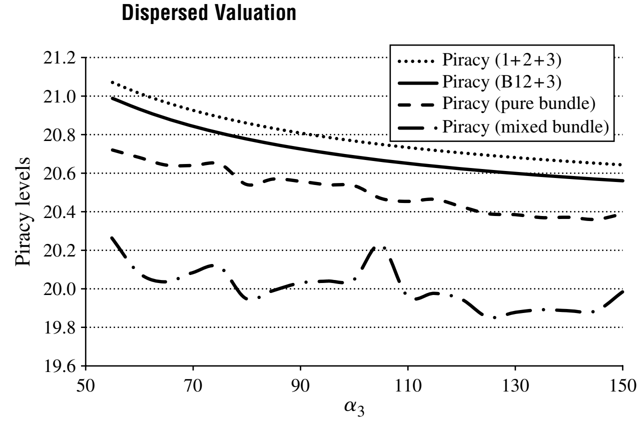


Figure 8 Total Piracy with Pure and Mixed Bundling When the Third Product Has Very High Valuation

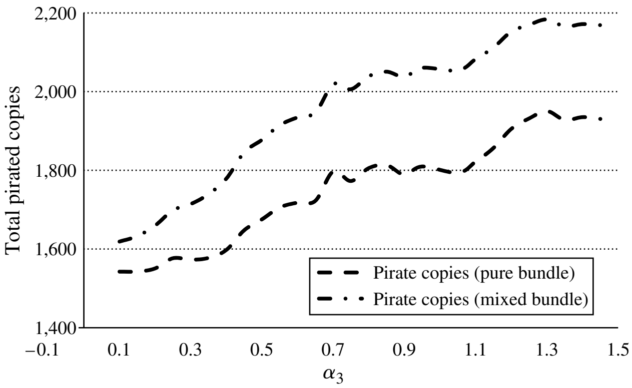
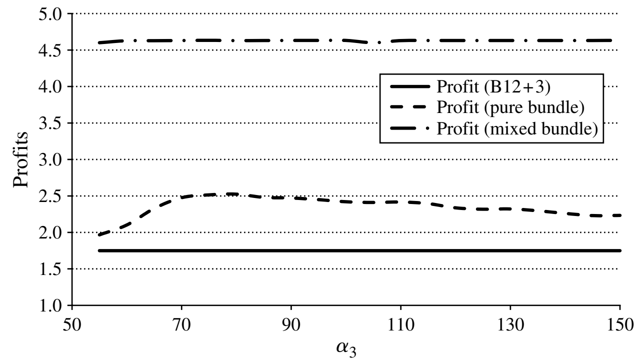


Figure 9 Relative Bundle Profits When the Third Product Has Relatively Dispersed Valuation



the individual piracy levels of the products when the products are relatively closer in their valuations. Another interesting revelation from Figure 7 is that beyond a certain point (as the valuation for product 3 decreases), there is relative little change in the piracy rates in general. Another interesting aspect is a jump in piracy rates with mixed bundling when $\alpha_3 > 100$; this coincides with the case when the third product has a smaller value than the first product and, therefore, the bundle between first product and third product is no longer even at the boundary of congruent piracy in the mixed-bundle setting. However, the overall piracy rates remain lower than that of all other scenarios.

The piracy levels presented in Figures 3 and 7 should not be confused with the total level of piracy in the market. Whereas, in the case of mixed bundling, the price impact of piracy reduces the piracy levels for each individual bundle significantly as opposed to the case when the bundle is sold as a pure bundle, the total piracy may go up because of increased sales volume and market coverage. As discussed earlier, figures present average piracy level per legitimate copy of the product sold. However, as Figure 6 demonstrates, the total coverage in the market increases almost threefold when products are offered in mixed bundles. This means that although average piracy per product sold decreases, because the sales are much larger, the total piracy in the market could increase. Figure 8 shows the total piracy for the case when the value of the third product in the bundle is high (i.e., by combining the results presented in Figures 3 and 6 to compute total piracy). As the figure shows, mixed bundling results in increased overall piracy due to increased market coverage.

Figure 9 shows the relative profits (logarithmic scale) of the pure-bundling and mixed-bundling environments as compared to the unbundled products. The results indicate an interesting trend that the relative profit levels of the mixed bundle remain relatively stable and the highest among all the bundling options. On the other hand although pure bundling stays in congruent piracy region, it shows an inverted U-shaped curve with respect to declining value of product 3. The relative profits with pure bundling first increase as the value of the third product decreases and then show steadily declining profits.

5. Concluding Remarks

The literature on the economics of bundling is vast, and the literature on the economics of piracy of digital goods is gaining momentum. These two strands of research, however, have not cross-pollinated. We address this lacuna by developing an analytical model that captures the mutual impacts of piracy and bundling and their collective impact on software pricing, seller profits, and consumer welfare. The most prevalent mode of software piracy, end-user piracy where several individuals together share a legal copy of the software, exhibits a form of bundling wherein a unit of the product is sold to a bundle of consumers. We develop a model that introduces a product bundle to this environment.

The traditional economic rationale of demand pooling from bundling that enables sellers to extract higher surplus and its particular attractiveness for information goods with negligible marginal and bundling costs carry over to our analysis (Bakos et al. 1999). In addition, the presence of piracy injects several new facets to our analysis. Bundling creates a shared level of piracy of disparate products, and under certain conditions to the detriment of one of the products. By construction of the copyright laws and by the letter of the law, the act of bundling itself can have a deterrence effect. This deterrence effect, along with shared piracy of products and demand pooling are ingredients that together dictate the overall piracy, pricing, profit, and welfare outcomes. Our analysis reveals several interesting insights. Bundling can be profitable even when the very act of bundling increases the piracy level of one of the products in the bundle. Termed *phantom piracy*, this represents a situation where sellers trade off higher piracy for one product in favor of lower piracy for the other product while deriving overall higher profits. Extensive simulation analysis shows that the region of phantom piracy is vastly expanded when additional products are introduced to the bundle. Price discounts that are typically offered by bundling are sharply deepened when piracy enters the equation. When piracy is a phenomenon to contend with, product bundling always increases consumer surplus even in scenarios where the seller may not realize higher profits. Unlike other forms of price discrimination that are often viewed by consumers with a jaundiced eye as they attempt to extract additional surplus from the consumers, product bundling in the software context can be a win-win scenario for both the buyers and sellers.

At first blush, our findings appear to run counter to the views commonly expressed with bundling in the related industry of music. Music, like software, is also a digital product that suffers from the problem of piracy. The practice of bundling—in the form of albums—has always been viewed as a practice that is not customer friendly. With the advent of online music, consumers appear to increasingly shun albums in favor of purchasing music à la carte. However, music albums are not bundles in the strictest sense of economics. The relative valuations of the songs that should be compiled into an album and the price discounts that theory suggests should be offered are often not found in music albums. Perhaps the problems rest not with bundling itself but rather how the industry has chosen to constitute the bundles. Bhattacharjee et al. (2009) suggest that "playlists"—bundles of songs compiled with a thematic purpose—can be a useful approach to propel consumers to purchase music in bundled formats.

Our analysis casts a fairly large shadow on the antitrust debate that bundling, in particular by monopolists such as Microsoft, garners. The practice of bundling Microsoft products, coupled with predatory pricing, is seen as a clear evidence of intent to harm competitors. It is often claimed that Microsoft bundling and pricing practices "chill future innovation and eliminate competition" (Olsen 1998). Missing in this debate is that an insidious threat—piracy—haunts the software industry. Unchecked and uncontrolled, piracy could potentially chill the economic incentive to innovate. And piracy does not discriminate; it harms not just a single firm but any and all of its competitors that create valuable intellectual property. Our analysis reveals that bundling and aggressive pricing (through bundle discounts) are an effective practice to combat piracy and it comes with the additional benefit that consumer surplus is increased as a consequence. Admittedly, bundling and aggressive pricing may in fact arise from anticompetitive intent, but intent itself does not constitute irrevocable evidence. Injecting the issue of piracy and its consequences into the debate may lead to a more reasoned and balanced analysis on acceptable practices by software companies.

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Appendix. Proofs

Proof of Proposition 1

First, let us derive conditions for $n_2^* > n_b^*$. From Equations (8) and (13), we have

$$\frac{N+\alpha_2 g(c,1)}{4\alpha_2 g(c,1)} > \frac{N(2\alpha_1+\alpha_2)+2\alpha_1\alpha_2 g(c,2)}{8\alpha_1\alpha_2 g(c,2)}.$$

Substituting with $\alpha_1 = \beta\alpha_2$ and simplifying, we obtain

$$2\beta g(c,2) > (2\beta+1)g(c,1).$$

Rearranging, we have $g(c,2)/g(c,1) > (2\beta+1)/(2\beta)$.

Now consider $n_1^* + n_2^* > 2n_b^*$. This can be written as

$$\frac{N+\alpha_1 g(c,1)}{4\alpha_1 g(c,1)} + \frac{N+\alpha_2 g(c,1)}{4\alpha_2 g(c,1)} > 2\frac{N(2\alpha_1+\alpha_2)+2\alpha_1\alpha_2 g(c,2)}{8\alpha_1\alpha_2 g(c,2)}.$$

Simplifying and rearranging we get

$$\frac{g(c,2)}{g(c,1)} > \frac{2\beta+1}{\beta+1}.$$

Proof of Proposition 2

The condition $n_1^* > n_b^*$ can be written as

$$\frac{N+\alpha_1 g(c,1)}{4\alpha_1 g(c,1)} > \frac{N(2\alpha_1+\alpha_2)+2\alpha_1\alpha_2 g(c,2)}{8\alpha_1\alpha_2 g(c,2)}.$$

By substituting $\alpha_1 = \beta\alpha_2$, we obtain

$$\frac{N+\beta\alpha_2 g(c,1)}{4\beta\alpha_2 g(c,1)} > \frac{N\alpha_2(2\beta+1)+2\beta\alpha_2^2 g(c,2)}{8\beta\alpha_2^2 g(c,2)}.$$

Further simplification yields $g(c,2)/g(c,1) > \beta + 1/2$.

Proof of Proposition 3

$$n_1^* = \sqrt{\frac{p_1^*}{g(c,1)}} \quad \text{and} \quad n_2^* = \sqrt{\frac{p_2^*}{g(c,1)}} \quad \text{and} \quad \eta_b = \sqrt{\frac{p_1^*+p_2^*}{g(c,2)}}.$$

From the above, we have

$$\eta_b = \sqrt{\frac{g(c,1)}{g(c,2)}}\sqrt{(n_1^*)^2+(n_2^*)^2}.$$

From Equations (8) and (13),

$$n_b^* = \frac{g(c,1)}{2g(c,2)}\left(2n_2^* + n_1^* - \frac{3}{4}\right).$$

The piracy reduction due to bundling (without price discounting) can be expressed as

$$n_1^* + n_2^* - 2\sqrt{\frac{g(c,1)}{g(c,2)}}\sqrt{(n_1^*)^2 + (n_2^*)^2}.$$

Similarly, the piracy reduction due to price discounting can be expressed as

$$2\sqrt{\frac{g(c,1)}{g(c,2)}}\sqrt{(n_1^*)^2 + (n_2^*)^2} - \frac{g(c,1)}{g(c,2)}\left(2n_2^* + n_1^* - \frac{3}{4}\right).$$

From the above expressions, it is easy to verify that the piracy reduction due bundling is increasing in $g(c,2)/g(c,1)$ and the piracy reduction due to price discounting is decreasing in $g(c,2)/g(c,1)$.

Proof of Proposition 4

From Equations (9) and (15) we need to show

$$\frac{[N(2\alpha_1+\alpha_2)+2\alpha_1\alpha_2 g(c,2)]^2}{32\alpha_1^2\alpha_2} > \frac{N^2(\alpha_1+\alpha_2)+4N\alpha_1\alpha_2 g(c,1)+\alpha_1\alpha_2(\alpha_1+\alpha_2)g(c,1)}{8\alpha_1\alpha_2}.$$

Substituting $\alpha_1 = \beta\alpha_2$, expanding and simplifying we get

$$\begin{aligned} &N^2 + 4\beta^2 N^2 + 4\beta N^2 + 4\beta^2\alpha_2^2 g^2(c,2) \\ &\quad + 8\beta^2 N\alpha_2 g(c,2) + 4\beta N\alpha_2 g(c,2) \\ &> 4\beta N^2 + 4\beta^2 N^2 + 16\beta^2 N\alpha_2 g(c,1) \\ &\quad + 4\beta^3\alpha_2^2 g(c,1) + 4\beta^2\alpha_2^2 g(c,1). \end{aligned}$$

Canceling $4\beta N^2 + 4\beta^2 N^2$ on both sides, rearranging and substituting α_2 for $\beta\alpha_2$ we get

$$\begin{aligned} &N^2 + 4\alpha_1^2 g^2(c,2) + 4N\alpha_1 g(c,2) - 4\alpha_1^2 g(c,1) \\ &\quad > \beta(4\alpha_1^2 g(c,1) + 8N\alpha_1(2g(c,1) - g(c,2))). \end{aligned}$$

Rearranging, we get the bound on β

$$\beta < \frac{N^2 + 4\alpha_1^2 g^2(c,2) + 4N\alpha_1 g(c,2) - 4\alpha_1^2 g(c,1)}{4\alpha_1^2 g(c,1) + 8N\alpha_1(2g(c,1) - g(c,2))}.$$

Proof of Proposition 5

From Equations (6) and (14), the prices of the individual products and the bundle in the presence of piracy, when $g(c,1) = c$ and $g(c,2) = 2c$, are given by

$$p_1^* = \left(\frac{N+\alpha_1 c}{4\alpha_1 c}\right)^2 c, \quad p_2^* = \left(\frac{N+\alpha_2 c}{4\alpha_2 c}\right)^2 c, \quad \text{and}$$
$$p_b^* = \left(\frac{N(2\alpha_1+\alpha_2)+4\alpha_1\alpha_2 c}{16\alpha_1\alpha_2 c}\right)^2 2c.$$

And the corresponding piracy levels (from Equations (5) and (13)) are given by

$$n_1^* = \frac{N+\alpha_1 c}{4\alpha_1 c}, \quad n_2^* = \frac{N+\alpha_2 c}{4\alpha_2 c}, \quad \text{and}$$
$$n_b^* = \frac{N(2\alpha_1+\alpha_2)+4\alpha_1\alpha_2 c}{16\alpha_1\alpha_2 c}.$$

For piracy of product 1 to exist $n_1^* > 1$. Otherwise, the consumer would simply purchase the product legally in which case $(p_1^*)_{\text{piracy}} = (p_1^*)_{\text{no-piracy}}$. Therefore, the boundary condition for product 1 can be expressed as

$$\frac{N+\alpha_1 c}{4\alpha_1 c} > 1.$$

Equivalently, the condition can be stated as $c < N/(3\alpha_1)$. Similarly, the appropriate boundary conditions for product 2 and the bundle are $c < N/(3\alpha_2)$ and $c < N(2\alpha_1+\alpha_2)/(12\alpha_1\alpha_2)$. Because $\alpha_1 \geq \alpha_2$, $N/(3\alpha_1) \leq N/(3\alpha_2)$.

Consider first the case when $N/(3\alpha_1) < N(2\alpha_1+\alpha_2)/(12\alpha_1\alpha_2)$. As long as $c < N/(3\alpha_1)$, all three products (the two individual products and the bundle) are pirated and the optimal prices are as shown above.

Employing Equation (6), (14), and (19), we have

$$\left(\frac{p_b^*}{p_1^*+p_2^*}\right)_{\text{piracy}} = \frac{(N(2\beta+1)+4\beta\alpha_2 c)^2}{8[(N+\alpha_2\beta c)^2 + (\beta N + \alpha_2\beta c)^2]}.$$

We can show that $\partial[(p_b^*/(p_1^*+p_2^*))_{\text{piracy}}]/\partial c > 0$ when $2\beta^2 - 3\beta + 5 \geq 0$. Because $\beta \geq 3/2$, the result follows.

Now consider the case when $N(2\alpha_1+\alpha_2)/(12\alpha_1\alpha_2) < N/(3\alpha_1)$. As long as $c < N(2\alpha_1+\alpha_2)/(12\alpha_1\alpha_2)$, all three products will be pirated as the results previous will hold. Now consider the case where $N(2\alpha_1+\alpha_2)/(12\alpha_1\alpha_2) < c < N/(3\alpha_1)$. In this case, the two individual products are pirated but not the bundle. In this case,

$$\left(\frac{p_b^*}{p_1^*+p_2^*}\right)_{\text{piracy}} = \frac{(p_b^*)_{\text{no-piracy}}}{(p_1^*+p_2^*)_{\text{piracy}}}.$$

Because the bundle price is not impacted by c (as there is no piracy of the bundle) and p_1^* and p_2^* are both monotonically decreasing in c, the result follows.

Proof of Proposition 6

From Equation (18), we have

$$\left(\frac{p_b^*}{p_1^*+p_2^*}\right)_{\text{no-piracy}} = \frac{2\beta+1}{2\beta+2}.$$

Proposition 5 shows that $(p_b^*/(p_1^*+p_2^*))_{\text{piracy}}$ is increasing in c when $c < N/(3\alpha_1)$. To prove the proposition, we need to show that $(p_b^*/(p_1^*+p_2^*))_{\text{no-piracy}} > (p_b^*/(p_1^*+p_2^*))_{\text{piracy}}$ when $c = N/(3\alpha_1)$.

Consider first the case when $N/(3\alpha_1) < N(2\alpha_1+\alpha_2)/(12\alpha_1\alpha_2)$. As long as $c < N/(3\alpha_1)$, all three products (the two individual products and the bundle) are pirated and thus we have

$$\left(\frac{p_b^*}{p_1^*+p_2^*}\right)_{\text{piracy}} = \frac{(N(2\beta+1)+4\beta\alpha_2 c)^2}{8[(N+\alpha_2\beta c)^2 + (\beta N + \alpha_2\beta c)^2]}.$$

Substituting $c = N/(3\alpha_1)$ in the above expression and simplifying yields

$$\left(\frac{p_b^*}{p_1^*+p_2^*}\right)_{\text{piracy}} = \frac{(7+6\beta)^2}{8[16+(1+3\beta)^2]}.$$

For $\beta \geq 1$, it can be shown that

$$\frac{2\beta+1}{2\beta+2} > \frac{(7+6\beta)^2}{8[16+(1+3\beta)^2]}.$$

Now consider the case when $N(2\alpha_1+\alpha_2)/(12\alpha_1\alpha_2) < N/(3\alpha_1)$. At $c = N/(3\alpha_1)$ we have

$$\left(\frac{p_b^*}{p_1^*+p_2^*}\right)_{\text{piracy}} = \frac{(p_b^*)_{\text{no-piracy}}}{(p_1^*+p_2^*)_{\text{piracy}}}.$$

To prove the proposition, we simply need to show that $(p_1^*+p_2^*)_{\text{piracy}} > (p_1^*+p_2^*)_{\text{no-piracy}}$. This result follows as $(p_1^*)_{\text{piracy}} > (p_1^*)_{\text{no-piracy}}$ and $(p_2^*)_{\text{piracy}} > (p_2^*)_{\text{no-piracy}}$.

Derivation of Consumer Welfare

The consumer welfare from a single software product can be defined as

$$W_1^T = (W_1^A - p_1)\, d_1,$$

where W_1^T is the total consumer welfare, W_1^A is the average system welfare, p_1 is the effective price paid by each consumer, and d_1 is the effective demand for the product. Note that W_1^A can simply be calculated as the average value of in the range $(p_i, N/\alpha)$ (see Figure 1). The conditional distribution in this range can be calculated as

$$f(x \mid x \geq p_i) = \frac{\alpha_i/N}{1-\alpha_i p_i/N} = \frac{\alpha_i}{N-\alpha_i p_i}. \tag{22}$$

We can now calculate the average consumer welfare as

$$\begin{aligned} W_1^A &= \int_{p_i}^{N/\alpha} x f(x \mid x \geq p_i)\, dx \\ &= \frac{\alpha_i}{N-\alpha_i p_i}\left[\frac{x^2}{2}\right]_{p_i}^{N/\alpha} \\ &= \frac{\alpha_i}{2(N-\alpha_i p_i)}[N^2/\alpha^2 - p_i^2], \end{aligned} \tag{23}$$

or, after simplification,

$$W_1^A = \frac{N+\alpha_i p_i}{2\alpha_i}. \tag{24}$$

Now because demand $d = N - \alpha_i p_i$

$$W_1^T = \left[\frac{N+\alpha_i p_i}{2\alpha_i} - p_i\right](N-\alpha_i p_i) = \frac{(N-\alpha p_i)^2}{2\alpha}. \tag{25}$$

Now because $p_i = (N-\alpha c)/(2\alpha) \Rightarrow N = 2\alpha p_i + \alpha c$, substituting the value of N in Equation (25) gives

$$W_1^T = \frac{(\alpha p_i + \alpha c)^2}{2\alpha} = \frac{\alpha(p_i+c)^2}{2}. \tag{26}$$

For the two products the total welfare

$$W_1^T + W_2^T = \frac{\alpha_1(p_{1i}+c)^2 + \alpha_2(p_{2i}+c)^2}{2}. \tag{27}$$

For the bundled product, we first calculate the cumulative density function $F(p_{bi})$ with respect to p_{bi}, an individual's effective bundle price:

$$F(p_{bi}) = \frac{1}{2}\frac{N}{\alpha_1}\frac{\alpha_2}{N} + \frac{\alpha_2}{N}\left(p_{bi} - \frac{N}{\alpha_1}\right) = \frac{2\alpha_1\alpha_2 p_{bi} - \alpha_2 N}{2\alpha_1 N} \tag{28}$$

$$= \frac{\alpha_2}{2\alpha_1} + \frac{\alpha_2 p_{bi}}{N} - \frac{\alpha_2}{\alpha_1}$$

$$= \frac{\alpha_2 p_{bi}}{N} - \frac{\alpha_2}{2\alpha_1} = \frac{2\alpha_1\alpha_2 p_{bi} - \alpha_2 N}{2\alpha_1 N}, \tag{29}$$

where p_{bi} is an individual's component of bundle price.

Or, the cumulative probability that a customer has a value greater than the effective individual price p_{bi} is

$$\begin{aligned} P(x \geq p_{bi}) &= 1 - \frac{2\alpha_1\alpha_2 p_{bi} - \alpha_2 N}{2\alpha_1 N} \\ &= \frac{N(2\alpha_1+\alpha_2) - 2\alpha_1\alpha_2 p_{bi}}{2\alpha_1 N}. \end{aligned} \tag{30}$$

Let the numerator of the above equation be represented by $D = N(2\alpha_1+\alpha_2) - 2\alpha_1\alpha_2 p_{bi}$. We can now calculate the conditional probability distribution in the range $(p_{bi}, N_1/\alpha_1 + N_2/\alpha_2)$ as

$$f(x \mid x \geq p_{bi}) = \begin{cases} \dfrac{\alpha_2/N}{D/(2\alpha_1 N)} = \dfrac{2\alpha_1\alpha_2}{D}, & \text{for } p_{bi} \leq x \leq N/\alpha_2; \\ \dfrac{(\alpha_1+\alpha_2)/N}{D/(2\alpha_1 N)} - \dfrac{\alpha_1\alpha_2 x/N^2}{D/(2\alpha_1 N)} = \dfrac{2\alpha_1(\alpha_1+\alpha_2)}{D} - \dfrac{2\alpha_1^2\alpha_2 x}{ND}, & \text{for } N/\alpha_2 \leq x \leq N/\alpha_1 + N/\alpha_2. \end{cases} \tag{31}$$

Therefore, the average system welfare is given by

$$W_b^A = \int_{p_{bi}}^{N/\alpha_1+N/\alpha_2} x f(x \mid x \geq p_{bi})\, dx \tag{32}$$

$$= \int_{p_{bi}}^{N/\alpha_2} x\frac{2\alpha_1\alpha_2}{D}\, dx + \int_{N/\alpha_2}^{N/\alpha_1+N/\alpha_2} x\left[\frac{2\alpha_1(\alpha_1+\alpha_2)}{D} - \frac{2\alpha_1^2\alpha_2 x}{ND}\right] dx \tag{33}$$

$$= \frac{2\alpha_1}{D}\left[\frac{\alpha_2}{2}\left(\frac{N^2}{\alpha_2^2} - p_{bi}^2\right) + \frac{\alpha_1+\alpha_2}{2}\left(\frac{N}{\alpha_1}\left(\frac{N}{\alpha_1}+\frac{2N}{\alpha_2}\right)\right) - \frac{\alpha_1\alpha_2}{3N}\left(\frac{N}{\alpha_1}+\left(\frac{N^2}{\alpha_1^2}+\frac{N^2}{\alpha_1\alpha_2}+\frac{N^2}{\alpha_2^2}\right)\right)\right]. \tag{34}$$

Because

$$\begin{aligned} d_b &= N\left(1+\frac{\alpha_2}{2\alpha_1} - \frac{\alpha_2}{N}p_{bi}\right) \\ &= \frac{N(2\alpha_1+\alpha_2) - 2\alpha_1\alpha_2 p_{bi}}{2\alpha_1} = \frac{D}{2\alpha_1} \quad \text{and} \quad \alpha_1 = \beta\alpha_2, \end{aligned}$$

the total welfare $W_b^T = (W_b^A - p_b)\cdot d_b$ can be written as

$$W_b^T = \frac{N^2-\alpha_2^2 p_{bi}^2}{2\alpha_2} + \frac{N}{\beta\alpha_2}\left[\frac{\beta\alpha_2+\alpha_2}{2}\left(\frac{N}{\beta\alpha_2}+\frac{2N}{\alpha_2}\right) - \frac{\beta\alpha_2^2}{3}\left(\frac{N}{\beta^2\alpha_2^2}+\frac{N}{\beta\alpha_2^2}+\frac{N}{\alpha_2^2}\right)\right] - \frac{N(2\alpha_1+\alpha_2)-2\alpha_1\alpha_2 p_{bi}}{2\alpha_1}p_{bi}. \quad (35)$$

Now because $p_{bi}^* = (N(2\alpha_1+\alpha_2) - 4\alpha_1\alpha_2 c)/(4\alpha_1\alpha_2)$ implies $N(2\alpha_1+\alpha_2) = 4\alpha_1\alpha_2 p_{bi} + 4\alpha_1\alpha_2 c$, substituting for $N(2\alpha_1+\alpha_2)$ in the subtrahend of Equation (35), we get

$$W_b^T = \left[\frac{N^2-\alpha_2^2 p_{bi}^2}{2\alpha_2} + \frac{N^2}{\beta\alpha_2}\left[\frac{\alpha_2(\beta+1)}{2}\frac{2\beta+1}{\beta\alpha_2} - \frac{\beta\alpha_2^2}{3}\frac{\beta^2+\beta+1}{\beta^2\alpha_2^2}\right]\right] - \frac{2\alpha_1\alpha_2 p_{bi}+4\alpha_1\alpha_2 c}{2\alpha_1}p_{bi} \quad (36)$$

$$= \frac{N^2-\alpha_2^2 p_{bi}^2}{2\alpha_2} + \frac{N^2}{\beta^2\alpha_2}\frac{4\beta^2+7\beta+2}{6} - \alpha_2 p_{bi}^2 - 2\alpha_2 c p_{bi}. \quad (37)$$

Rearranging N^2 and p_{bi}^2 terms we get

$$W_b^T = \frac{N^2}{2\alpha_2} + \frac{N^2}{\beta^2\alpha_2}\frac{4\beta^2+7\beta+2}{6} - \frac{\alpha_2 p_{bi}^2}{2} - \alpha_2 p_{bi}^2 - 2\alpha_2 c p_{bi} \quad (38a)$$

$$= \frac{N^2(7\beta^2+7\beta+2)}{6\beta^2\alpha_2} - \frac{3\alpha_2 p_{bi}^2 + 4\alpha_2 c p_{bi}}{2}. \quad (38b)$$

Substituting $N = 2\alpha_1 p_{1i} + \alpha_1 c$ in the first terms of Equation (40b),

$$W_b^T = \frac{\alpha_1^2(2p_{1i}+c)^2(7\beta^2+7\beta+2)}{6\beta^2\alpha_2} - \frac{3\alpha_2 p_{bi}^2 + 4\alpha_2 c p_{bi}}{2}. \quad (38c)$$

Because $\alpha_1 = \beta\alpha_2$, we have

$$W_b^T = \frac{\alpha_1(2p_{1i}+c)^2(7\beta^2+7\beta+2)}{6\beta} - \frac{3\alpha_2 p_{bi}^2 + 4\alpha_2 c p_{bi}}{2}. \quad (38d)$$

Proof of Proposition 7

To prove $W_b^T > W_1^T + W_2^T$, we compare Equations (38d) and (27). We need to show

$$\alpha_1(2p_{1i}+c)^2(7\beta^2+7\beta+2) - 9\beta\alpha_2 p_{bi}^2 - 12\beta\alpha_2 c p_{bi} > 3\beta\alpha_1(p_{1i}+c)^2 + 3\beta\alpha_2(p_{2i}+c)^2. \quad (39a)$$

Because $\alpha_1 = \beta\alpha_2$, we get

$$(2p_{1i}+c)^2(7\beta^2+7\beta+2) - 9\beta\alpha_2 p_{bi}^2 - 12\beta\alpha_2 c p_{bi} > 3\beta(p_{1i}+c)^2 + 3(p_{2i}+c)^2. \quad (39b)$$

Now,

$$p_{2i}+c = \frac{N-\alpha_2 c}{2\alpha_2} + c = \frac{N+\alpha_2 c}{2\alpha_2}. \quad (40a)$$

Substituting $N = 2\alpha_1 p_{1i} + \alpha_1 c$ and $\alpha_1 = \beta\alpha_2$,

$$p_{2i}+c = \frac{2\alpha_1 p_{1i} + \alpha_1 c + \alpha_2 c}{2\alpha_2} = \frac{2\beta p_{1i} + (\beta+1)c}{2}. \quad (40b)$$

Substituting for $p_{2i}+c$ in Equation (41b), the right-hand side of the inequality becomes

$$3\beta(p_{1i}^2 + 2p_{1i}c + c^2) + 3\frac{(4\beta^2 p_{1i}^2 + 4\beta(\beta+1)p_{1i}c + (\beta+1)^2 c^2)}{4} \quad (41a)$$

$$= \frac{1}{4}[12\beta p_{1i}^2 + 24\beta p_{1i}c + 12\beta c^2 + 12\beta^2 p_{1i}^2 + 3\beta^2 c^2 + 6\beta c^2 + 3c^2 + 12\beta^2 p_{1i}c + 12\beta p_{1i}c] \quad (41b)$$

$$= \frac{1}{4}[12\beta^2 p_{1i}^2 + 3\beta^2 c^2 + 12\beta^2 p_{1i}c + 12\beta p_{1i}^2 + 36\beta p_{1i}c + 18\beta c^2 + 3c^2]. \quad (41c)$$

Expanding the left-hand side in inequality (39b), we get

$$28\beta^2 p_{1i}^2 + 7\beta^2 c^2 + 28\beta^2 p_{1i}c + 28\beta p_{1i}^2 + 28\beta p_{1i}c + 7\beta c^2 + 8p_{1i}^2 + 8p_{1i}c + 2c^2 - 9p_{bi}^2 - 12p_{bi}c. \quad (42)$$

We need to prove that "expression (42) > expression (41c)," i.e.,

$$112\beta^2 p_{1i}^2 + 28\beta^2 c^2 + 112\beta^2 p_{1i}c + 112\beta p_{1i}^2 + 112\beta p_{1i}c + 28\beta c^2 + 32p_{1i}^2 + 32p_{1i}c + 8c^2 - 36p_{bi}^2 - 48p_{bi}c > 12\beta^2 p_{1i}^2 + 3\beta^2 c^2 + 12\beta^2 p_{1i}c + 12\beta p_{1i}^2 + 36\beta p_{1i}c + 18\beta c^2 + 3c^2. \quad (43)$$

Regrouping we get

$$100\beta^2 p_{1i}^2 + 25\beta^2 c^2 + 100\beta^2 p_{1i}c + 100\beta p_{1i}^2 + 76\beta p_{1i}c + 10\beta c^2 + 32p_{1i}^2 + 32p_{1i}c + 5c^2 - 36p_{bi}^2 - 48p_{bi}c > 0. \quad (44)$$

Now $N = 2\alpha_1 p_{1i} + \alpha_1 c$ and from $N(2\alpha_1+\alpha_2) = 4\alpha_1\alpha_2 p_{bi} + 4\alpha_1\alpha_2 c$ we get $N = (4\alpha_1\alpha_2(p_{bi}+c))/(2\alpha_1+\alpha_2)$. Hence,

$$\alpha_1(2p_{1i}+c) = \frac{4\alpha_1\alpha_2(p_{bi}+c)}{\alpha_2(2\beta+1)}$$

$$\Rightarrow\ 2p_{1i}+c = \frac{4p_{bi}+4c}{2\beta+1} \quad (45)$$

$$\Rightarrow\ (2p_{1i}+c)(2\beta+1) = 4p_{bi}+4c$$

$$\Rightarrow\ p_{bi} = \frac{4p_{1i}\beta + 2\beta c + 2p_{1i} - 3c}{4}. \quad (46)$$

Substituting for p_{bi} in inequality (44) and simplifying the left-hand side into a quadratic polynomial in β, we get

$$(64p_{1i}^2 + 64p_{1i}c + 16c^2)\beta^2 + (64p_{1i}^2 + 64p_{1i}c + 13c^2)\beta + 23p_{1i}^2 + 35p_{1i}c + \tfrac{83}{4}c^2 > 0, \quad (47)$$

true for any positive values of p_{1i}, c, and β.

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