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Double Backsolve Remains Unsupported

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This article initially examines the mechanics of the established option pricing method (OPM) backsolve (OBS). It then quickly moves to a critical analysis of the more recently developed double backsolve (DBS) method, which certain practitioners have proposed as an alternative to OBS. We review the literature cited to support DBS and find it does not, in fact, support its use. In addition, we note some inconsistencies in the current use of DBS. We conclude that until better arguments are proffered or superior methods are developed, practitioners should continue to use OBS, and reviewers should continue to reject valuations relying on DBS.

Introduction

The AICPA's 2004 practice aid on the *Valuation of Privately-Held-Company Equity Securities Issued as Compensation* (subsequently updated in 2013, AICPA 2013; hereafter termed "Guide") introduced the option pricing method (OPM) to a wide audience. Since that time, the OPM has gained considerable traction in the valuation community and has been used to allocate company value to individual securities when multiple classes of equity are outstanding. The OPM has also been used to "backsolve" for the value of entire companies based on preferred stock financings.

In recent years, the authors have witnessed a proliferation of a substitute method for the OPM backsolve (OBS), the so-called "double backsolve" (DBS). While OBS is supported by widespread acceptance from valuation experts and auditors[1] and has the backing of the Guide,[2] DBS has none of these characteristics.

Valid criticisms of OBS have been made, but DBS is subject to the same criticisms. The authors of one of the studies cited in critiquing the OBS anticipate and overcome the objection that supposedly necessitates the DBS. In addition, it is observed that some practitioners switch from using DBS in early financing rounds to OBS in later financing rounds. The arguments that practitioners offer in rejecting OBS in early rounds would apply to later rounds as well, making their practice inconsistent and illogical.

OBS is an elegant method, the primary downside of which is its complexity. While better methods may be developed in the future, the even more complex DBS is not such an improvement and should continue to be rejected by reviewers.

The next section of this article summarizes the mechanics of OBS and DBS. This is followed by a discussion of the arguments offered in support of DBS. Finally, we show the inadequacy of such arguments, including an inconsistent application of them.

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Mechanics of OBS

Research by Fisher Black, Myron Scholes, and Robert Merton on option pricing theory stands as a landmark in mathematical finance. Together the authors developed a pioneering formula, known as the Black-Scholes formula, designed to formulaically value stock options. Prior to Black-Scholes, option pricing was empirically, not rationally, estimated (Bouchaud and Sornette 1994). Although the Black-Scholes formula was originally conceived to compute the value of a European call option, Black, Scholes, and Merton's method has been applied to value a wider set of derivative securities, contingent claim contracts, corporate liabilities, guarantees, and investment decisions (The Royal Swedish Academy of Sciences 1997).

The OPM uses Black-Scholes to value securities by modeling their values as functions of the total equity value. If the company is worth x when shareholders liquidate, Security α will be worth $y = f(x)$ at that time. $f(x)$ is typically a continuous[3] piecewise function, with the

[1] See, for example, PricewaterhouseCoopers (2019) and Puca (2020).

[2] See, for example, the following sections of the Guide: 4.07, 4.16, 6.30–6.41, and Chapter 9.

[3] Jump discontinuities create challenges in the application of the OPM.

number of "pieces" being determined by the company's capital structure and the liquidation, dividend, and conversion rights of each share class.

The value $f(x)$ can be modeled using a portfolio of options that can, in turn, be valued using Black-Scholes. Therefore, if we have assumptions for the five inputs to Black-Scholes (strike price, value of the underlying asset, risk-free rate, time to maturity, and volatility) we can determine the values of the options, $f(x)$, and, by extension, the security. It is also true that if we know the value of a security and have four of the five assumptions, we can solve for the 5th. In this way, we can solve for the value of the underlying asset (the equity) if we already know the value of a class of preferred stock. This method of company valuation is OBS.

A key assumption of the Black-Scholes model and, by consequence, OBS, is that stock returns are normally distributed, and the value of the underlying security at maturity therefore follows a lognormal distribution. In the case of OBS, the assumption is that the equity value is lognormally distributed. Consequently, the exit values relative to invested capital are lognormally distributed. In his bestseller *The Black Swan,* Nassim Taleb argued that while many important natural phenomena are normally distributed, others are not (Taleb 2010). For example, if equity returns were normally distributed, the stock market crash of 1987—which caused the S&P 500 Index to drop 20.5% in a single day—should happen less frequently than once every trillion years (Chandrashekaran 1998). The market recognizes this and prices options systematically differently from Black-Scholes, resulting in volatility "smiles" and "sneers" (Dumas, Fleming, and Whaley 1998).

Despite these limitations, Black-Scholes is backed by Nobel Prize–winning research and is the cornerstone of option pricing (Wilmott, Howison, and Dewynn 1995).

Evolution of DBS

Clients typically desire the lowest value possible for common stock in so-called "409A valuations." One firm authoring these valuations started to model lower values of common stock than OBS implies by adding a second "scenario" to its analysis. The equity value in the second scenario was assumed to be exactly equal to the value of the company's invested capital. With the value of invested capital typically equaling the sum of preferred stock liquidation preferences, preferred receives all of its money back in this scenario, and common has no value. This effectively cut the value of common stock by whatever weight was placed on the scenario. Place 50% weight on the scenario, and the value goes down by 50%.

The idea was as elegant as an arbitrary haircut. Reviewers routinely questioned the assumption of a large probability of an exit value exactly equal to the value of the capital invested. Such an assumption would necessarily violate the underlying OBS assumption of lognormality of returns. Were there data to support this? There were not, and the model was routinely rejected.

After facing so many rejections, some practitioners attempted to put "lipstick on the pig" and added complexity to the new scenario. Instead of assuming an exit at exactly the value of invested capital, they said that the equity value in the scenario should, on average, equal the invested capital but should be distributed lognormally.[4] The practitioners would run two OPMs, with the first reflecting an earlier liquidity scenario and the second reflecting a later liquidity scenario. A shorter time to liquidity (and sometimes lower volatility) was used for the early scenario. By selecting a short time horizon in the "fail" scenario, the practitioners effectively kept the distribution very tightly centered around the invested capital value. As a consequence, most of the value remained with preferred in this scenario, and very little went to common, achieving the same desired result. This added complexity did nothing to address the question of why one would assume that the equity value in the added scenario would equal the value of the invested capital.

Not surprisingly, given this ignominious background, no academic literature has been proffered in support of DBS. However, in audit responses DBS proponents argue that it is appropriate to violate the assumption of lognormal equity returns because returns to early-stage venture investments are not lognormally distributed.

Theoretical versus actual return distributions

If a variable is lognormally distributed, this means the log of the variable is normally distributed, following the familiar bell curve distribution. Any variable x that has a lognormal distribution can take any value between zero and infinity. As shown in Figure 1, a lognormal distribution is skewed such that the mean, median, and mode are all different. The probability density function on the vertical axis illustrates how likely the variable x is to have a certain outcome. Note that the lognormal distributions shown in Figure 1 assume a mean of zero. As the standard deviation increases from 0.4 to 0.8, the lognormal distribution widens and flattens such that a broader range of outcomes becomes more probable.

DBS proponents have proffered two studies of the actual returns of venture investments and compared these to the distributions implied by OBS. One source cited by proponents of DBS is a book by Andrew Metrick and

[4]The equity value in the other scenario is solved in such a way that the weighted value of the new preferred in both scenarios equals its purchase price, hence the name "double backsolve."

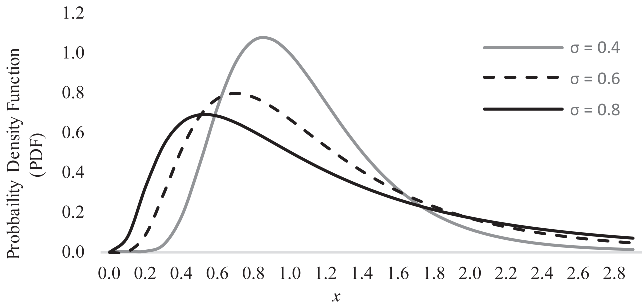

Figure 1
Lognormal Distribution (μ = 0)

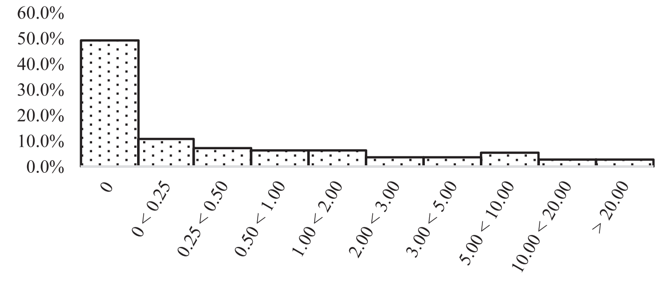

Figure 2
Metrick and Yasuda (2011) Implied Distribution of Exit Returns (First-Round Investments)

Ayako Yasuda (2011) in which the authors studied venture capital (VC) exit returns prior to 2001 (Fig. 2). The authors used data from Sand Hill Econometrics that tracked VC investments through exit. After making some assumptions regarding the underlying data, the authors found that across first funding rounds, 49% of exits resulted in zero returns to investors. Twenty-five percent of exits resulted in returns (gross value multiples, or GVMs)[5] higher than 0 but below 1.0×. Ten percent of exits resulted in returns between 1.0× and 3.0×. Finally, the percentage of exits returning 3.0× and above was 16%.

DBS proponents also refer to another study by James Walling and Cindy Moore (2010). Walling and Moore analyzed early-stage exit multiples across the software, medical devices and equipment, and biopharmaceuticals industries between 2001 and 2008. Walling and Moore found that 42% to 63% of exits resulted in GVMs of zero across the three industries analyzed. Eight percent to 13% of exits resulted in GVMs higher than zero but below 1.0×, while 9% to 27% of exits resulted in GVMs between 1.0× and 3.0×. And between 15% and 23% of exits resulted in GVMs higher than 3.0×. Figures 3 and 4 show the data for the software industry.

OBS will model non-zero GVMs in all cases. As can be seen in Figures 2 and 3, these studies found such a return in approximately half of the investments. These studies could therefore provide a legitimate critique of OBS. While Black-Scholes has the support detailed previously and the OPM maintains a status as a default method, the authors recognize that better methods could be developed. If matching real-world GVMs was determined to be of paramount importance, appraisers would have to decide which real-world data they would be trying to match among many dimensions (e.g., time period, data source, industry, stakeholder).

It is also worth mentioning that in the very same book with the referenced studies, Metrick and Yasuda anticipate the argument against lognormality: "The argument is this: 'VC investments do not have returns like public companies. Instead, a VC investment often returns absolutely nothing, and occasionally returns a huge amount. This binary type of outcome is not consistent with a lognormal distribution.'" However, the authors conclude, "Although the assumptions of the Black-Scholes approach do not hold for private companies, the Black-Scholes solution still seems to be an unbiased approximation for the value of options in these companies."

So the studies do not closely match the GVMs implied by OBS. But do they match those implied by DBS? Hardly. It should be obvious that were one to try to match the distributions in the studies, one's model would have to result in roughly half of the GVMs being zero. Yet, like OBS, DBS models *no* chance of a zero return. Instead, as mentioned earlier, it models a large number of GVMs at 1.

Just like the OPM and OBS implied distributions, the implied distribution from DBS varies based on the inputs used (company capitalization, probability for each scenario, term for each scenario, and volatility differences in the scenarios—if any). However, we can generally note

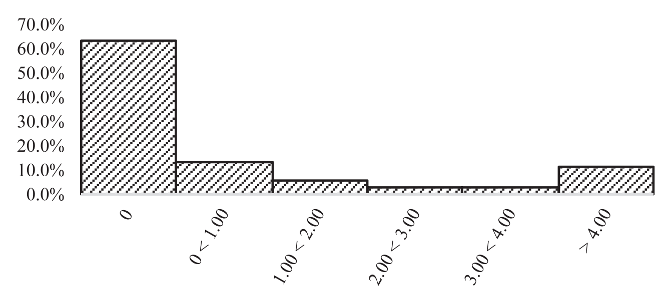

Figure 3
Walling and Moore (2010) Implied Distribution of Exit Returns (Software Industry 2001–2008)

[5]While Metrick and Yasuda define GVMs as the return on preferred stock, the data they rely on treat preferred stock as common stock in calculating returns.

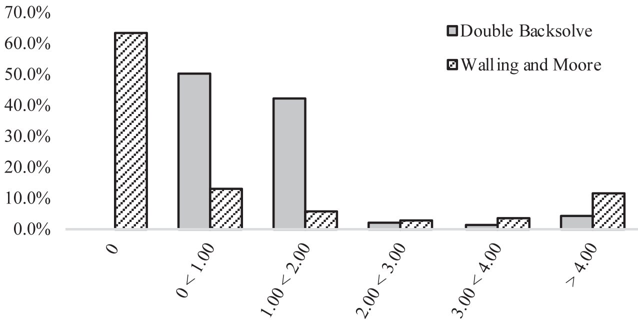

Figure 4
DBS Implied Distribution Versus Walling and Moore (2010) Implied Distribution of Exit Returns (Software Industry 2001–2008)

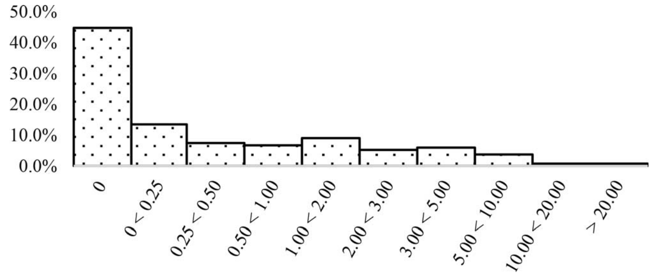

Figure 5
Metrick and Yasuda (2011) Implied Distribution of Exit Returns (Third-Round Investments)

that because the "early exit scenario" assumes an equity value at invested capital and a short time to liquidity, it will imply a large number of outcomes around the invested capital value.

Using assumptions from an example posted on the Internet, the implied GVM distributions from a DBS are modeled below. The "early exit scenario" is weighted using a probability of 75% and includes a volatility of 15%, a time to maturity of one year, and a risk-free rate of 1.53%. The "late exit scenario" is weighted using a probability of 25% and includes a volatility of 80%, a time to maturity of five years, and a risk-free rate of 1.55%. The modeled capitalization structure includes (1) five million outstanding shares of Series Seed Preferred Stock with an original issue price of $1.00, a 1× liquidation preference, no participation rights, and a 1:1 conversion ratio, (2) 10.3 million outstanding shares of Common Stock, and (3) 3.2 million outstanding Common Stock options with a strike price of $0.25.

It is clear from the chart above that DBS GVMs do not match those from the studies referenced (they are not even close), so it is odd that these studies would be offered in support of its use.

(In)consistency

We have heard some practitioners argue that DBS is only appropriate for early-stage companies. These practitioners use the DBS for Series A companies and OBS for Series C companies. However, the below data from Metrick and Yasuda on third-round investments demonstrate that their GVMs follow a similarly shaped distribution to those of the first-round investments (Fig. 5). If one were to use DBS for first-round investments based on the study data, the same reasoning would require the use of DBS in third rounds as well.

Conclusions

- OBS is based on Nobel Prize–winning theory and a distribution that is used to price options and other financial instruments in the real world. In addition, OBS has become institutionalized in the Guide.
- Like any method, OBS has strengths and weaknesses. The criticism that real-world GVMs do not match those implied by OBS is a valid one.
- To support their critique of the Black-Scholes assumption of lognormality, DBS proponents cite an Andrew Metrick study. Yet Metrick is no supporter of their viewpoint. The opposite is true. In the very same book in which Metrick published this study, Metrick anticipated this critique of Black-Scholes and dismissed it. Metrick explained why he still uses Black-Scholes despite problems with the assumption of lognormality.
- DBS also doesn't match the cited GVMs, not by a mile. It also requires several additional assumptions, increasing its complexity. While some of OBS assumptions are difficult to verify, those of DBS are basically unauditable.
- The arguments that practitioners offer in rejecting OBS in early rounds would apply to later rounds as well. It is inconsistent and illogical for practitioners to switch from using DBS in early financing rounds to OBS in later financing rounds.
- There is no guidance suggesting the use of DBS, nor is there logical or academic support for DBS. Until better arguments are proffered or superior methods are developed, practitioners should continue to use OBS, and reviewers should continue to reject valuations relying on DBS.

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