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Predicting Bidders' Willingness to Pay in Online Multiunit Ascending Auctions: Analytical and Empirical Insights

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We develop a real-time estimation approach to predict bidders' maximum willingness to pay in a multiunit ascending uniform-price and discriminatory-price (Yankee) online auction. Our two-stage approach begins with a bidder classification step, which is followed by an analytical prediction model. The classification model identifies bidders as either adopting a myopic best-response (MBR) bidding strategy or a non-MBR strategy. We then use a generalized bid-inversion function to estimate the willingness to pay for MBR bidders. We empirically validate our two-stage approach using data from two popular online auction sites. Our joint classification-and-prediction approach outperforms two other naïve prediction strategies that draw random valuations between a bidder's current bid and the known market upper bound. Our prediction results indicate that, on average, our estimates are within 2% of bidders' revealed willingness to pay for Yankee and uniform-price multiunit auctions. We discuss how our results can facilitate mechanism-design changes such as dynamic-bid increments and dynamic buy-it-now prices.

Key words: online auctions; predicting willingness to pay; dynamic-mechanism design
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1. Introduction and Background

Online auctions exemplify the Internet's ability to become a temporally and spatially unconstrained market maker. Yet, while expanded in scale and scope, Internet auctions can arguably be considered to be lacking in the skills of an expert auctioneer. An expert auctioneer can be credited with maintaining the temporal order of the auction and the movement of bids (Smith 1990). While critical to auction outcomes, this feature has not been incorporated in online auction design and has been paid scant attention in the literature. This motivates us to ask:

(a) Can we develop an analytical prediction model to estimate, in real time, the willingness to pay (WTP) of bidders participating in online multiunit uniform-price and Yankee-price auctions?

(b) Can we test the efficacy of the prediction empirically using data from online auction sites?

We believe that a real-time WTP estimation model is a necessary first step in proposing mechanism-design changes such as dynamic-bid increments, commonly implemented by expert auctioneers, in high-volume mechanized online auctions. We demonstrate how an online auctioneer, equipped with an estimation model such as ours, can use dynamic-bid increments to achieve higher revenue and allocation efficiency, as well as set dynamic buy-it-now prices.

From a theoretical perspective, progressive ascending multiunit auctions have received only limited attention, usually under a set of assumptions that do not hold up in the online context. For example, bidders are assumed to be homogeneous, typically described as being symmetric, risk-neutral, and adopting Bayesian-Nash equilibrium strategies. While tenable in the context of face-to-face single-item auctions, this set of assumptions readily breaks down in the majority of multiunit online auctions. For such auctions, it is well known that the computation of equilibrium bidding strategies is intractable (Nautz and Wolfstetter 1997).

Our approach relies on the enhanced information acquisition and computational potential of today's Internet auctions. It makes use of microlevel bidding data and assumes that bidders conform to a myopic best-response (MBR) bidding strategy that ties bidders' revealed bids to their underlying WTP. MBR bidding agents' bids conform to an equilibrium strategy with the assumption that the agents view the current bidding round as the last of the auction and take prices as given (Parkes 2001). Recent work (Bapna et al. 2004) on bidding strategies in Yankee auctions indicates that approximately 66% of the serious bidders conform to this strategy. A serious bidder was defined as a bidder whose highest bid was equal to at least 70% of the final auction price. Other bidders use a non-MBR strategy where they typically reveal their WTP in the first bid they place.

We develop a two-stage nested approach that, in the first stage, classifies bidders' strategies, followed by an estimation technique that predicts bidders' WTP. The classification model identifies bidders as either adopting an MBR or a non-MBR strategy. Then, for bidders using an MBR strategy, we develop an analytical model to predict their WTP. We empirically validate our approach using data from two different types of ascending auctions, showing that our joint classification-and-prediction approach outperforms two other naïve prediction strategies that draw random valuations between a bidder's current bid and the known market upper bound. We can estimate, on average, within 2% of bidders' revealed WTP for Yankee and uniform-price multiunit auctions.

In §2, we review the literature. In §3, we provide insights into the market mechanism we investigate and introduce our multiunit auction data set. Section 4 focuses on the bidder strategy-classification scheme and the development of an analytical model for predicting MBR bidders' WTP. Prediction accuracy is tested empirically in §5. Section 6 demonstrates the utility of WTP estimation in inferring the final price prior to the close of the auction, and on setting dynamic-bid increments as well as dynamic buy-it-now prices. Section 7 provides conclusions.

2. Related Literature

The literature on auctions is vast (see, for example, McAfee and McMillan 1987, Milgrom and Weber 1982, Milgrom 1989, Rothkopf and Harstad 1994, Menezes 1996). Recent literature on online auctions is comprehensively summarized in a recent survey (Ockenfels et al. 2006).

Many online auctions use progressive open multiunit formats, which have the following benefits over their sealed-bid counterparts (Cramton 1998): (i) efficiency of the price-discovery process; (ii) revenue maximization; (iii) reduction of the winners curse; and (iv) privacy and implementation. Ausubel (2004) proposes an ascending-bid auction for multiple units that ameliorates the demand-reduction incentive by progressively and iteratively increasing the asking price, but without showing how auctioneers should determine the increments of the ask price. The price-increment aspect has implications on auction efficiency and revenue. We posit that accurate prediction of bidders' WTP can form the basis of dynamically determining optimal asks. Carare (2001) demonstrates the utility of working with microdata, observable in online auctions, by deriving marginal valuations of bidders in a focused market for computer central processing units, with a goal to recover distributions of valuations for computer processors. Our work also relies on transient within-auction data but, in contrast, attempts to model bidding behavior for real-time predictive purposes for a broad spectrum of products sold through both uniform-price and Yankee auctions. Both auction mechanisms are widely used in the business-to-consumer (B2C) online market. uBid.com and Samsclub.com are representative popular sites.

Using the Paris Bourse as a test bed, Biais et al. (1999) examined the accuracy of valuation information derived from preopening market trade games. Their study shows that, although the information derived from such games is noisy in the early stages of the game, there is some convergence to true market values as the market opening time approaches. This approach is similar to ours, predicting bidder WTP in open ascending-price auctions. The utility of valuation prediction has also been recognized in the artificial intelligence field, where automated agents use value-discovery models as components of bidding agents. Parkes and Ungar (2000) use the notion of MBR bidding strategies among agents to illustrate how proxy bidders that embrace this strategy can be shielded from manipulation.

Another study (Plott and Salmon 2004) uses a surplus-maximization strategy to describe bidding behavior in simultaneous ascending auctions. Although the auction mechanism is different from the one studied by Parkes and Ungar (2000), the notion of MBR strategies is used as a way of tying bidders' iterative type revelation to their WTP.

For a real-time prediction-and-calibration approach to be applicable, it is first necessary to understand the space of bidding strategies used by the bidders. We can then use the information available during the course of an auction to identify a given bidder's strategy accurately. In this context, we justify our assumption of bidders adopting an MBR strategy in multiunit online auctions based on Bapna et al. (2001, 2004), which identify different bidding strategies adopted in Yankee auctions, two of which (used by approximately 66% of all bidders) are consistent with the MBR definition.

Bapna et al. (2002) study multiunit auctions and provide a strategy with which an auctioneer can maximize her gains and empirically approximate bidder behavior as we do here. The specific problem solved in Bapna et al. (2002) is to compute a fixed optimal bid increment for an auction to maximize the expected revenue given the tail distribution of highest $N+1$ bidders, where N is the number of units on sale. In contrast, in this paper, we estimate the distribution of bidders' WTP by individually estimating every bidder's maximum WTP. Thus, the technique developed here can be used to estimate the highest $N+1$ bidders' WTP. These can be seen as a proxy for valuations and can be used to maximize the seller's revenue using the model described in Bapna et al. (2002). However, applications of the WTP prediction model developed here are broad and not limited to computation of optimal bid increments. While Bapna et al. (2002) yield fixed-bid increments for the auctioneer, we show in §6 how an auctioneer could develop dynamic-bid increments. Other unique applications are the ability to infer a lower bound on the final auction price during the early stages of an auction and also to establish dynamic buy-it-now prices.

3. Progressive Online Multiunit Auctions

We deal with two popular online auction mechanisms in the wider B2C category of auctions: Yankee and uniform-price progressive multiunit auctions. These mechanisms offer consumers multiple units of the same item. Bidders compete for the items, with each bidder submitting a bid indicating the quantity desired and the per-unit price he is willing to pay. These auctions are conducted in an open format and bidders can see the bids of competing bidders. Bidders can join the auction at any time during the auction duration, typically several days. The auctioneer spells out the rules that govern the bidding activity.

All bidders are expected to submit bids that are at least as high as the *minimum required bid*. The *bid increment* is the minimum increment by which a bidder must exceed the minimum winning bid to win an item. Bidders are not bound to bid strictly following the bid increment. As noted by Easley and Tenorio (2004), jump bidding is often observed in Yankee auctions. Auction sites usually specify an *auction closing time*; however, some auctions sites extend the auction duration if bidding activity is observed in the last few minutes of the auction. Another common feature of online auctions is the *suggested retail price*, or a *buy-it-now price*. Essentially, these variables cap the expected auction revenue because rational bidders will not bid beyond the suggested retail price. This is validated empirically. In no case, in our data set of 787 uniform-price online auctions, did the final bid exceed 90% of the suggested retail price. Thus, bidders' WTP is capped by the suggested retail prices. In §3.1, we provide further details of our data set.

3.1. Online Auction Data Collection

Our analysis uses data from two multiunit online auction websites (Samsclub.com and uBid.com). We deployed automatic auction-tracking agents to observe and collect data on entire auction proceedings. One auction site uses a uniform-price auction mechanism, while the other uses a Yankee auction mechanism. Our automated agent collected bidding data from 787 uniform-price auctions and 205 Yankee auctions, for a total of 78,014 bids or bid revisions. The auction-tracking agent was programmed to visit the identified online auction's Web pages in intervals of 5 to 15 minutes, take snapshots of the auction, and record the bidding history of the auction site. With this technique, we were able to maintain a complete history of the auction, noting each of the bids submitted and revisions made by each bidder in the auctions we tracked. Sample validation was done based on Bapna et al. (2003a, b).

3.2. Description of the Data

Data on the uniform-price auctions represent 90 products, mainly electronics. The Yankee auctions data set contains electronics and computing goods. The average number of bidders per auction in the uniform-price auctions is close to 36 (standard error = 2.26), while the Yankee auctions on average attracted 47 (standard error = 4.35) bidders per auction. The average lot size in both Yankee and uniform-price auctions is 13. On average, bidders submit 1.3 (0.01) and 1.83 (0.05) bids for the uniform-price and Yankee auctions, respectively. The range of number of bid revisions is 9 and 38 for the uniform-price and Yankee auctions, respectively. Appendix 1 in the Online Supplement to this paper (available at http://joc.pubs.informs.org/ecompanion.html) provides details of the variables collected as well as summary statistics of some key variables.

4. Prediction Model—Myopic Best-Response Strategy

We begin with the model for the MBR bidding strategy, which can be interpreted as a surplus-maximizing bid calculated by a bidder in a given round of the auctions assuming that all competing bids remain unchanged from the previous round. As new arrivals come in, and bidders get displaced from the winning list, the myopic assumption allows for belief revision by the same bidder to account for additional information. This results in a revision of the bidder's WTP each time a bidder revises her bid.

Consider an auction for N units of an item. Let the current winning bids be denoted by $x_1, x_2, \ldots, x_N$, in increasing order of magnitude. Bidders submitting these bids are assumed to have WTP values $W_1, W_2, \ldots, W_N$, respectively, bounded below by the bids already submitted, i.e., $W_1 \geq x_1, W_2 \geq x_2, \ldots, W_N \geq x_N$. When a new bid b is received, it must be greater than x_1, which is displaced from the winning list. We assume that b was determined to optimize myopically the expected surplus that the bidder will derive from the auction. The MBR strategy maximizes a bidder's expected surplus, given already-submitted bids and a belief of the actual WTP values of bidders who submitted the earlier bids. The belief regarding other bidders' WTPs is a probability distribution with support in the range of the lowest winning bid and an upper bound, which can be set to a publicly known price for the item being auctioned, such as the suggested retail price for the auction. The myopic approach allows for belief revision as the auction progresses, allowing us to capture the information signals of the new arrivals. Bidders who resubmit bids revise their initial beliefs about others' WTP values.

Let the bidder who submitted the new bid b have a WTP value W. Suppose that the new bid b is greater than k of the current winning bids. Therefore, the new sequence of winning bids is $x_2, x_3, \ldots, x_k, b, x_{k+1}, \ldots, x_N$. For the new bidder to win, given this state of the auction, at least one of the k bidders whose bids are smaller than b must have WTP less than b, assuming no new bidders join the auction.

Let the new bidder's belief about the WTPs of any of the current winners be an independent random variable with density function f_i, and a distribution function F_i, with support in the range $[x_1, m]$, where x_1 is the smallest winning bid, and m is an indicative fixed price for the item. We assume that F_i is continuous and twice differentiable over its support. Consistent with early auction studies (Milgrom and Weber 1982) and based on current online auction research studying similar auction types (Hidvégi et al. 2006, Pinker et al. 2003), we assume an independent private-values setting. Thus, the probability that at least one of the currently winning bidders has a WTP value that is less or equal to b is $1-\prod_{i=1}^{k}(1-F_i(b))$.

Let P_d and P_u be the prices paid by the new bidder in Yankee and uniform-price auctions, respectively. $P_d = b$, as each bidder pays a price equal to their bids in the Yankee auction. On the other hand, $x_1 < P_u \leq b$, and will be a function of the WTP of the k bidders who are outbid by the new bid and by the value of the new bid b itself.

If the auction uses a discriminatory-pricing scheme, where bidders pay a price equal to their bids, the new bidder will enjoy an expected gain

$$E(G) = (W-b)\left(1-\prod_{i=1}^{k}(1-F_i(b))\right), \tag{1}$$

and the expected gain in a uniform-price auction will be

$$E(G) = (W-P_u)\left(1-\prod_{i=1}^{k}(1-F_i(b))\right). \tag{2}$$

We assume that the observed bid b maximizes the expected gain (1) or (2), depending on the pricing scheme. Therefore, the observed bid should satisfy the first-order and second-order conditions for a maximum expected gain. Equations (3) and (4) show the first-order conditions for maximum expected gain under Yankee and uniform-pricing schemes, respectively.

$$\frac{\partial(E(G))}{\partial b} = -\left(1-\prod_{i=1}^{k}(1-F_i(b))\right) + (W-b)\left(\sum_{j=1}^{k} f_j(b)\prod_{i=1-\{j\}}^{k}(1-F_i(b))\right) = 0, \tag{3}$$

$$\frac{\partial(E(G))}{\partial b} = -\frac{\partial P_u}{\partial b}\left(1-\prod_{i=1}^{k}(1-F_i(b))\right) + (W-P_u)\left(\sum_{j=1}^{k} f_j(b)\prod_{i=1-\{j\}}^{k}(1-F_i(b))\right) = 0. \tag{4}$$

After observing bid b, and assuming that it was determined to maximize the bidder's expected surplus, we can make inferences about the corresponding WTP value. By solving (3) and (4) for W, we get the predicted WTP value of the bidder under the respective pricing scheme. The expressions for WTP value prediction are

$$\widehat{W}_{\text{Yankee}} = b + \frac{1-\prod_{i=1}^{k}(1-F_i(b))}{\sum_{j=1}^{k} f_j(b)\prod_{i=1-\{j\}}^{k}(1-F_i(b))}$$

and

$$\widehat{W}_{\text{Uniform}} = P_u + \frac{P_u'\left(1-\prod_{i=1}^{k}(1-F_i(b))\right)}{\sum_{j=1}^{k} f_j(b)\prod_{i=1-\{j\}}^{k}(1-F_i(b))}.$$

Because the distribution of WTP, of a new bidder, is bounded below by the minimum required bid and above by the suggested retail price, we use a left triangular distribution for empirical validation. This distribution is a special case of the beta distribution and, because it requires fewer parameters, is easier to estimate (Johnson 1997). Appendix 2 in the Online Supplement provides the specific analytical derivation of WTP values assuming a triangular distribution.

5. Empirical Validation of the Prediction Model

Our WTP estimation technique first classifies the bidders as conforming to an MBR strategy or not. For a non-MBR strategy, we estimate their WTP to be their current bid. We compute the WTP for MBR bidders using the prediction model in §4. We now present three alternative methods for MBR-strategy identification, and later the results of the prediction accuracy of the model.

5.1. Bidding-Strategy Classification into MBR and Non-MBR

To separate MBR from non-MBR bidders, we begin by examining the extant understanding of bidding strategies adopted by real-world online bidders. Bapna et al. (2004) used online auction data from 1999 and 2000 and found a stable taxonomy of bidder behavior containing five types of bidding strategies in Yankee auctions. Bidders pursue different strategies that, in aggregate, realize different winning likelihoods and consumer surplus. Bapna et al. (2005) extend this analysis to uniform-price auctions and find a remarkable consistency in the mix of bidding classes across the two auction types. Further, the robustness of the Bapna et al. (2004) classification is confirmed by Slavova's (2006) replication of the original classifications using a different sample of Yankee auctions.

We first examine how to determine whether a bidder is myopic. One indication of this would be if there were bid revisions made by the bidders, implying that at the time of bid placement, the bidders were relying on information available to them. Recall that MBR bidders conform to an equilibrium strategy with the assumption that the bidders view the current round as the last round of the auction and take prices as given (Parkes 2001). Bapna et al. (2004) indicate that while the participatory and opportunistic strategies conform to this behavior, the evaluatory strategy does not.

We first define two naïve rules. These are called *naïve* because both rely on only the response variable (observed bids) to undertake a classification. They do not use any other predictor attributes that we may know about the bidder in question or the auction itself.

Definition 1 (Basic MBR Classification Rule). Classify a bid that is lower than the largest winning bid X_N to be MBR, while a bid that exceeds the largest winning bid as non-MBR.

Using this classification method, 16% of the bids in our uniform-price auctions data set and 12% of the bids in our Yankee auctions data set would be classified as non-MBR.

It is also conceivable that determining a threshold that takes into account more information about the current winning bids would intuitively improve the strategy classification. Intuitively, a bid that is significantly higher than current winning bids, relative to the variance of the winning bids, does not conform to an MBR strategy.

Let μ and σ be the mean and standard deviation of winning bids. Suppose that b is the next bid that is submitted to the auction. Define $z = (b - \mu)/\sigma$, and let $z' = \sigma z/(x_1 - \mu)$ represent the corresponding truncated left-end measure that accounts for the fact that bids submitted must exceed the minimum winning bid.

Definition 2 (Normalized MBR Classification Rule). Classify a bidder with z' greater than an empirically determined cutoff as not conforming to the MBR strategy. All other bidders are MBR. The cutoff is arrived at by randomly partitioning the data into a training set (20% of the data) and a validation set (80%). The cutoff is chosen to minimize the overall misclassification rate in the training set, and the accuracy of the process is judged based on examining the confusion matrix of the validation set.

Note that if $x_1 = \mu$, i.e., when all the previous winning bids have the same value, then $\sigma = 0$ and z' is indeterminate. In such cases, Definition 2 cannot be used to classify a bid. While we did not encounter this condition empirically in our data set, when such a condition occurs, Definition 1 can be used to classify the bid.

We tested different cutoff points on the training set to get z' values with the highest classification accuracy. These values are 0.9 and 0.5 for the uniform-price and Yankee auctions, respectively. By applying these critical values to the remaining 80% of the validation data, we realized accuracy levels of 62% and 47% in predicting MBR bidders in uniform-price and Yankee auctions, respectively.

Next, we consider a more sophisticated rule that considers further auction attributes, in the form of independent variables, to enrich the classification scheme.

Definition 3 (Logistic-Regression Rule). Classify bidders into MBR and non-MBR based on the logit of their odds, derived from a multiattribute logistic-regression model.

5.1.1. Dependent Variable. The dependent variable has a binary value that shows if the bidder is using an MBR or non-MBR strategy. At each bidding instance, our prediction model estimates the consumer's WTP for a product. If the predicted WTP at a particular bidding instance is less than the final bid of a specific bidder, the dependent variable takes a value of one, indicating that the bid was constituted using an MBR strategy. Otherwise, we consider the bidder to be non-MBR.

5.1.2. Explanatory Variables. We consider variables for which an auctioneer can acquire data at the time of making a prediction on the bidders' WTP. The explanatory variables used in the logit model were based on the analytical model, as well as some observable strategic behavior or aggressiveness measures. From the analytical model, we can infer that the upper bound on the expected bid price for an item m and the lot size of the auctions N are likely to influence the adopted bidding strategy. These are captured in X_{1i} and X_{3i}, respectively, in (5), the best-fit logit regression model

$$\mathrm{Log}\left(\frac{P_i}{1-P_i}\right) = \alpha + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \beta_5 X_{5i} + \varepsilon_i, \qquad (5)$$

where

- P_i is the probability that the bidder is MBR;
- X_{1i} is the ratio K/N, where K is the number of current winning bids that are smaller than the new bid and N is the lot size;
- X_{2i} is the number of times the bidder has revised his bid;
- X_{3i} is the standard deviation of current winning bids;
- X_{4i} is the average of current winning bids;
- X_{5i} is the normalized elapsed auction time.

X_{1i} and X_{2i} capture the strategic behavior of the bidders. High values of X_{2i} would indicate that the bidder was a participator. A ratio close to 1 for X_{1i} would suggest an evaluatory non-MBR bidder.

Table 1 shows estimates of the model coefficients and their statistical significance for the uniform-price and Yankee auctions. All model variables, except X_{2i} in the uniform-price auctions, are significant in explaining the predicted classification. Appendix 3 in the Online Supplement, which presents correlation matrices for the variables used in the model, supports independence of the predictor variables.

The logit-classification model developed on the randomly chosen 20% training data yields a classification accuracy of 81% and 90% on the remaining 80% validation set for uniform-price and Yankee auctions, respectively. This is higher than both the *normalized* and the *basic* classification rule. Armed with the three classification schemes, we next seek to infer the WTP for both types of bidders.

Table 1 Strategy-Classification Model's Coefficient Estimates

	Uniform-price auctions				Yankee auctions			
Coefficient	Estimate	S.E.	Wald	Sig.	Estimate	S.E.	Wald	Sig.
β_1	−7.48	0.74	103.12	0.00	−9.98	0.50	398.30	0.00
β_2	−0.43	0.40	1.18	0.28	1.88	0.37	26.16	0.00
β_3	−0.05	0.03	5.03	0.03	−0.004	0	9.41	0.00
β_4	−0.02	0.01	9.31	0.00	−0.003	0	46.28	0.00
β_5	2.42	0.56	18.88	0.00	1.34	0.45	8.90	0.00
Constant	3.77	0.59	39.87	0.00	1.65	0.40	16.89	0.00

5.2. Accuracy of the WTP Prediction Model

Recall that for bidders classified as non-MBR, we estimate their WTP to be their current bid. For MBR bidders, we apply the model developed in §4. Table 2 shows the absolute mean percentage difference between WTP as predicted by our model and the actual WTP, as conservatively estimated by bidders' final bids. The results show the accuracy levels when the WTP model for MBR bidders is applied to all bidders (without any classification) and, subsequently, when the three strategy-classification models, the *basic*, the *normalized*, and the *multiattribute logit*, are used.

With no classification and considering all bidders (winners and losers), our predictions' mean absolute error is 20.43% and 9.9% in uniform-price and Yankee auctions, respectively. As we incorporate the strategy-classification schemes, the prediction error is reduced for uniform-price and Yankee auctions, with the multiattribute logit-classification rule yielding the smallest errors (2.6% and 1.8% error for uniform-price and Yankee auctions, respectively). Table 2 also indicates that the exclusion of winners reduces the prediction error in uniform-price auctions from 20.43% to 11.96%. A similar effect, albeit of a smaller magnitude, is observed among Yankee auctions.

We also sought to isolate and report the performance of our model on the set of bidders whose final bid was at least 50% of the final auction price. The motivation to isolate these bidders comes from the observation that there are many bidders who participate in the initial part of the auction when prices are very low and then drop out of the auction. Making predictions for these types of bidders results in an overprediction that may not be representative of the overall picture. When we isolated these early dropouts, we realized a 3.63% prediction error in uniform-price auctions and 10.74% for Yankee auctions.

5.3. Performance of the WTP Prediction Model and Alternative Solutions

Although, to the best of our knowledge, there are no dynamic models for predicting bidders' WTP with an approach similar to ours, some alternative and

Table 2 **Prediction Accuracy of Bidders' WTP**

	Absolute mean (standard deviation) of prediction error (%)					
	Uniform-price auctions			Yankee auctions		
MBR/non-MBR classification strategy	All bidders	Losers only	Bidders' with bids > 50% of price	All bidders	Losers only	Bidders' with bids > 50% of price
None	20.43 (40.69)	11.96 (21.29)	3.63 (14.83)	13.43 (24.03)	11.20 (33.65)	10.74 (12.81)
Basic	15.41 (37.62)	11.41 (21.08)	2.36 (14.18)	12.80 (23.83)	8.04 (31.52)	10.31 (12.82)
Normalized	8.48 (21.25)	7.79 (19.23)	2.45 (21.03)	7.89 (21.53)	6.34 (22.65)	7.19 (17.89)
Multiattribute logit model	2.63 (16.59)	2.31 (17.90)	1.89 (15.21)	1.89 (6.97)	2.03 (8.34)	2.16 (7.27)

fairly naïve approaches can be conceived. One potential method is to use the bids themselves as proxies for the WTP. We call this the *actual-bids proxy method*. Using this method, the auctioneer errs only by underpredicting the bidders' actual WTP. This method does not provide the auctioneer with any additional information that can be used to calibrate the auction process. It provides an absolute lower bound on the WTP.

Definition 4 (A Random-Draws Model). Assume that bidders' WTPs will be a random variable with support between the bid submitted and a known fixed price $[b, m]$. The random model takes independent draws, one for each bidder, from this distribution and assumes those to be their WTP.

For consistency and comparison purposes, we assume the same WTP distribution as we used in the empirical results above. This method has a two-sided risk of both overpredicting and underpredicting the actual WTP. It also lacks rationality, but certainly pulls a value from a feasible range.

Using our data set, we conduct a comparison of the predictions from these three models: our proposed model, actual-bids proxy, and the random-draws model. For uniform-price and Yankee auctions, the error in predicting the bidders' WTP from his first bid (using our method as well as the other two) is statistically greater than zero. Due to the large number of observations (4,752 for uniform-price and 3,872 for Yankee auctions), the power of the test is extremely high, and even minor deviations from the observed WTP results in rejection of the null hypothesis of equality.

Our prediction is statistically and qualitatively closest to the observed WTP compared with other approaches even with the first bids. As we move to second and third bids, our prediction starts making accurate WTP estimates as indicated by the p-values of paired t-tests in Table 3. However, the other two prediction methods result in rejection of the null hypothesis with high significance. This provides strong support for our method in comparison to other ad hoc prediction approaches.

To further examine the performance of our prediction model, we ask whether the actual predictions made from the different approaches are indeed significantly different. This approach offers a more direct comparison of the three approaches because they are not measured against the ex-post actual bid. The results provide overwhelming support to reject a hypothesis that equates predictions of our model to predictions from the naïve methods (p-values for the comparison between predictions were all less than 0.0001).

To gain further insight into the performance of our prediction model, we classify our prediction mean absolute error according to the auction duration and compare against the random model. The results are shown in Figures 1(a), 1(b), and 1(c). In both the Yankee and uniform-price cases, the random model has prediction errors that are significantly higher than our model in every time interval. Additionally, the significance of the difference between the model and random predictions of WTP increases over time (Figure 1(c)). This trend in noted for both Yankee and uniform-price auctions.

6. Applications of the Two-Stage Classification-Prediction Approach

6.1. Inference on Final Auction Revenue

It is logical to assume that if the predictions are accurate, an auctioneer should be able to infer a lower bound on the final auction revenue from the

Table 3 **p-Values for the Variation Between Predictions and Actual WTP for Alternative Prediction Methods**

		p-value for H_0: Predicted WTP = Actual WTP	
Prediction method	Comparison bid	Uniform-price auctions	Yankee auctions
Our prediction	1st bid	3.33279E−26	2.0827E−152
	2nd bid	0.001082154	0.078658
	3rd bid	0.798344	0.14566224
Current bid proxy	1st bid	4.99491E−20	1.00126E−29
	2nd bid	0.00090675	4.82E−18
	3rd bid	0.087145	7.8489E−08
Random prediction	1st bid	6.2769E−168	1.74903E−87
	2nd bid	4.18539E−23	0.0007637
	3rd bid	7.848E−05	2.69E−12

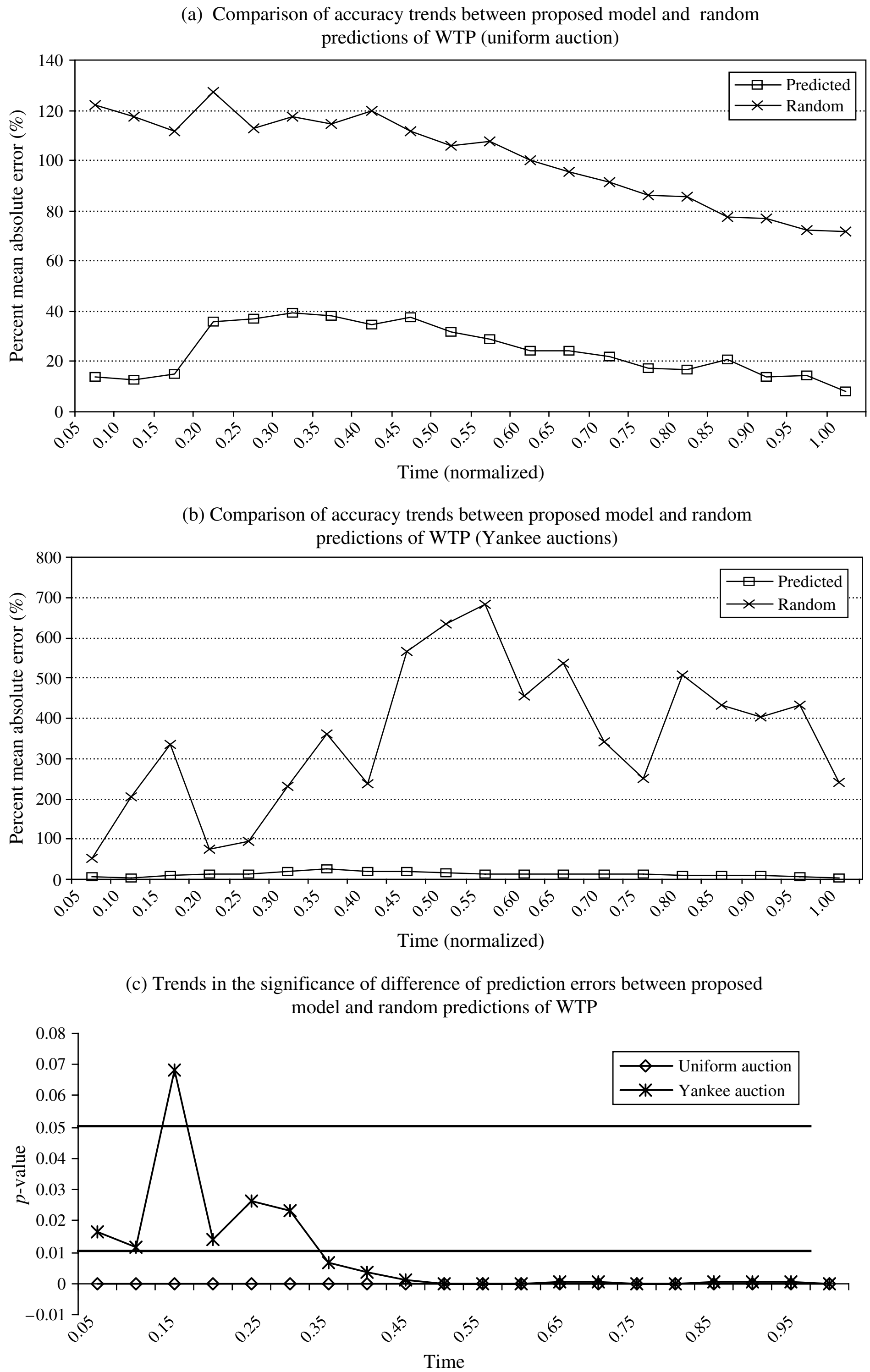

Figure 1 Comparison of Prediction Accuracy Between the Proposed WTP Prediction Model and a Random-Based Model

predicted WTPs. Such inference can be done prior to the close of the auctions, as soon as the number of bids exceeds the lot size offered. Conservatively, we can assume that the current estimates correspond to the final WTPs. Using the predicted WTPs at any stage of the auction, the predicted revenue for the uniform-price and the Yankee auctions, say R_u and R_D, respectively, are

$$R_u = NW_{(1)}, \tag{6}$$

$$R_D = \sum_{i=1}^{N} W_{(i)}, \tag{7}$$

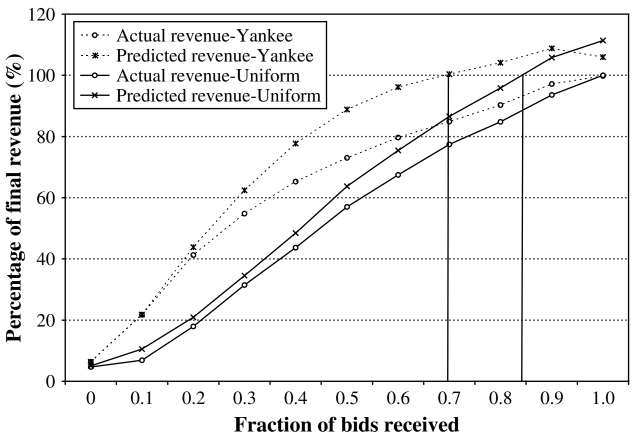

Figure 2 Percentage of Predicted and Auction Revenue to Final Auction Revenue

where $W_{(i)}$ represents the ith highest estimated WTP among the top N WTPs, at that time. The earlier such an estimate can be made accurately, the more utility it has for the auctioneer to use it to dynamically calibrate the mechanism.

Figure 2 displays a comparison of the progression of the actual auction revenue to the predicted revenue, as per (6) and (7), by the proportion of bids received. Observe that we are able to estimate, on average, the final auction revenue with just 85% of the bids that were submitted in uniform-price auctions and with just 70% of the bids that were submitted in the Yankee auctions.

When coupled with the enhanced computational capabilities inherent to Internet auctions, it is interesting to consider mechanism-design opportunities that rely on WTP estimation.

6.2. Dynamic-Bid Increments

While a full-blown analysis of how to use the estimation technique to determine dynamic-bid increments optimally is beyond the scope of this paper, we provide some intuition using an example.

In one of the auctions in our data set, a uniform-price auction was conducted for a power tool. This auction used a fixed-bid increment of one dollar. Using the same stream of bids as were received in the actual auction, we replicated the evolution of this auction.

Dynamic bid-increment rule. Let the minimum-bid increment be determined as the difference between the minimum winning bid value and the $(N\text{th}+1)$ highest predicted WTP.

The motivation behind this choice of a bid increment is to give bidders with the lowest WTP the incentive to bid at the earliest opportunity. In the long run, such an approach will increase the mechanism's allocative efficiency. In this particular auction, there were six units of the product, so the bid increment was set as the difference between the minimum winning bid value and the seventh-largest predicted WTP. In Figure 3, we show the variability in the computed minimum-bid increment. We also compare the revenue formation in the two auctions under the respective bid-increment-setting methods.

Figure 3(a) shows that, based on the predicted WTP values, the estimated minimum bid will differ from fixed-bid increments. Figure 3(b) shows marginal gain in revenue, realized through reallocations that are forced by the dynamic-bid increments. Interestingly, from the perspective of the social planner, these reallocations account for an increase in the allocative efficiency of the auction from 99.8% to 100%. Allocative efficiency measures to what extent the goods are allocated to the bidders that value them the most. This anecdotal example is meant to give the reader a flavor of our ongoing work in calibrating multiunit auctions in real time. Note that such changes in bidders' strategy may have an endogenous impact on auction dynamics, but investigation of such endogenous impact is beyond the scope of this paper.

6.3. Dynamic Buy-It-Now Prices

A cursory look at current online auctions reveals that auctioneers have modified their auction models to cater to buyers who are interested in a quick deal, in lieu of waiting for the auction to close. eBay calls this a *buy-it-now price*. At any time during the auction, the auctioneer offers the bidders a fixed-price offer, and bidders may opt to buy the product at that price instead of continuing to participate in the auction. In the case of single-item auctions, this terminates the auction. While similar features (namely, suggested retail prices) are offered on multiunit auctions, anecdotal evidence, presented in §1, suggests that the current implementation is not effective, in that a vast majority of auctions close below the suggested prices.

In setting a buy-it-now price, the auctioneer has to achieve a balance between setting a price that is too high to be effective and setting a price that is too low such that it results in lost expected revenue. We propose that auctioneers can use the predicted WTP to adjust buy-it-now prices dynamically in accordance with the bidders' demand functions for specific auctions. The key to buy-it-now prices should be to avoid cannibalization, i.e., not setting a price that is too low such that a person who will not win otherwise ends up buying. Our predictions allow auctioneers to tailor buy-it-now prices in accordance with their risk profile. These prices could range from a risk-seeking Nth–highest-valuation estimate to a more conservative highest-valuation estimate, as well as all the interim possibilities.

We also expect that dynamic buy-it-now prices will decrease the duration of multiunit auctions, a benefit that is already being reaped by single-unit eBay

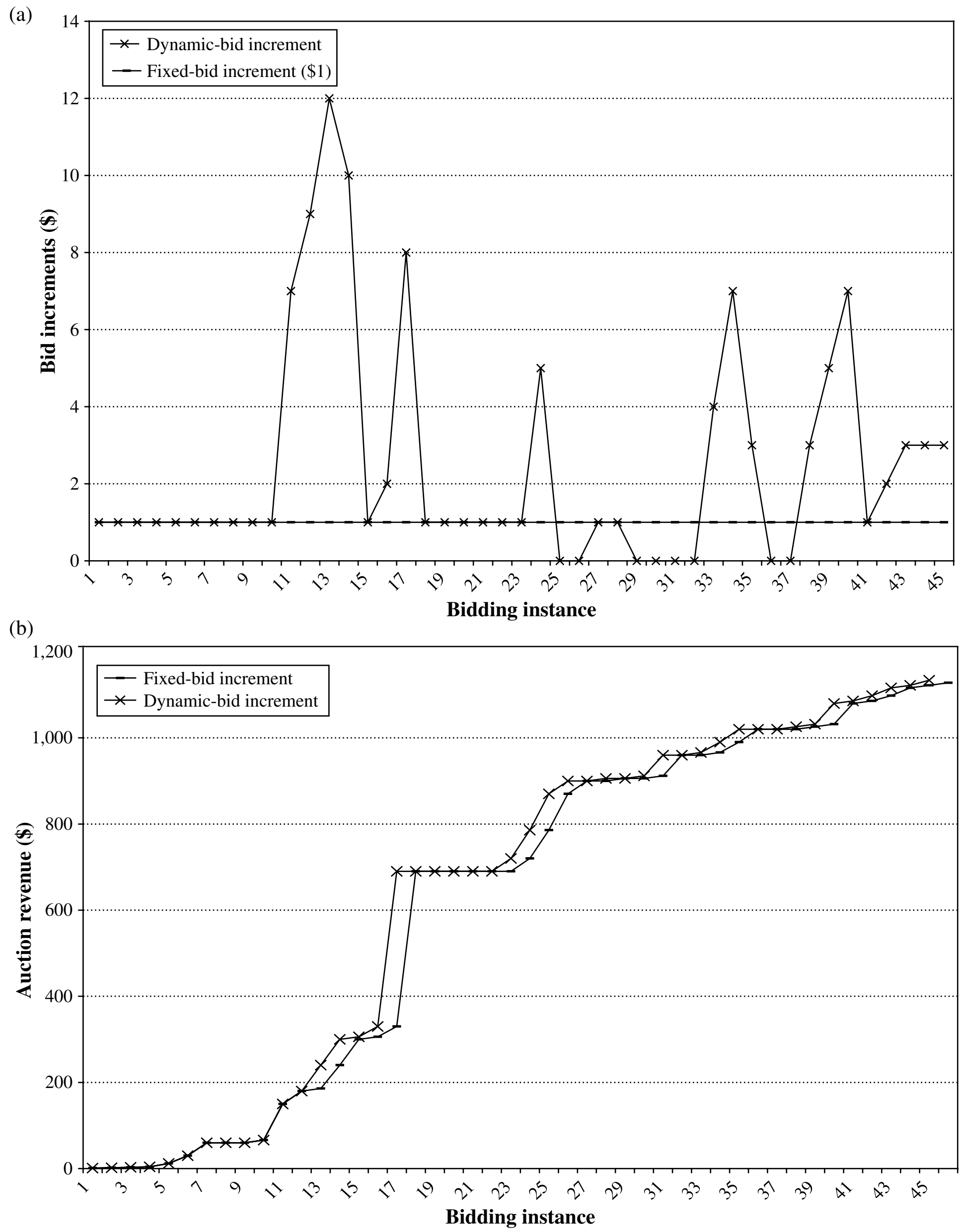

Figure 3 Comparison of Dynamic and Fixed-Bid Increment Setting—(a) Bid Increments, (b) Revenue

auctions that offer the buy-it now option. Setting optimal dynamic buy-it-now prices in multiunit settings remains a promising area of future work.

7. Conclusion and Future Research

Our work is motivated by the opportunity to bring back the skills of an expert auctioneer in the physical world, capturing by gut and feel the underlying essence of the auction room's WTP, into a high-volume, automated but computationally powerful online environment. We present a two-stage classification followed by a WTP estimation approach that performs well on real online bidding data. We are able to estimate, on average, within 2% of bidders' revealed WTP for a large number of online Yankee and uniform-price multiunit auctions. Our joint classification-and-prediction technique significantly outperforms alternative approaches that draw random valuations between a bidder's current bid and the known market upper bound.

Our findings enable inference on final auction prices prior to the close of the auction, which in turn can be used to make real-time mechanism-design changes to increase the auctioneer's revenue, maximize allocative efficiency, and potentially, through smart agents, bidders' surplus. We expect future research to find more interesting uses of the prediction model developed here. We believe that real-time value discovery tools, such as the one demonstrated in this paper, will provide the foundation for dynamically calibrating the online auction mechanism, to maximize the likelihood of obtaining desirable equilibria. They can also serve as building blocks for designing the next generation of smart bidding agents whose incentives are aligned with bidders.

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