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Judgment in Probabilistic Analysis

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Abstract

The chief use of probabilistic methods is to assess risk and opportunity, making them most applicable to situations of significant uncertainty. Hence, the cardinal sin unique to probabilistic analysis is to underestimate the range of outcomes. Unfortunately, the situations of the greatest uncertainty are also the ones where poor judgment is most likely to create unreliable results and dangerous decisions. The best judgment in probabilistic analysis is that which recognizes the full range of uncertainty by carefully framing the problem and by avoiding pitfalls which artificially reduce the range of results.

Introduction

Probabilistic analysis has become an essential tool[1] of the practicing reservoir engineer and reserve evaluator because its benefits are undisputed and well-documented.[2,3,4,5] They include the following:

- Probabilistic analysis forces the practitioner to think more completely, thoroughly, and thus clearly about the issues at hand.
- Probabilistic analysis can reveal the drivers of value or risk more clearly, making it possible to focus on risk mitigation efforts, data acquisition, further analysis and upside potentials.
- The results of a probabilistic study give a decision-maker more information about upside and downside uncertainty to inform his business decisions, future plans and portfolio analysis.
- Probabilistic analysis communicates the uncertainty unambiguously (to those conversant in the terms of statistics).

The first two uses add the greatest value. It is only in the framing, interrogation and audit of the model that the user obtains these advantages. Moreover, those benefits must be actively pursued in the process in order to obtain a meaningful quantitative result. Without judicious implementation of the model, the quantified results may mislead and endanger the decision-maker with unmerited confidence.

Much effort has been spent on discussion of input distributions to the probabilistic analysis, i.e., the form and range of the uncertain variables. Unfortunately, these considerations are dwarfed in importance by the architecture of the model. Model architecture represents the way the model is set up, e.g., the type of calculation, the number of parts, the correlation of parts, and the rules in the model. The discussion below deals first with issues related to model architecture and second with issues related to the input distributions.

In designing the architecture of a probabilistic model, it is essential to identify those drivers with the greatest impact on uncertainty, to consider all possible sources of uncertainty, to select an appropriate calculation methodology and level of detail.

Input distributions are defined by range and form. Definition of these two parameters, however, is predicated upon proper understanding of biases and types of uncertainty. All of these are discussed below.

Poor model-building causes an excessively narrow distribution of results and higher estimates of "reasonably certain" values. For example, the three most commonly cited pitfalls of implementation (aggregation, correlation, and range of input variables) all tend to reduce the range of outcomes. Ironically, though the high confidence end of the distribution is used to define Proved reserves, the extremes of a resultant distribution of outcomes are more poorly defined and subject to change than central estimates. Identifying the judgment calls which impact the range of results makes it possible to appreciate the limitations and subjectivity of probabilistic analysis.

Model Architecture

The single most important factor in the construction of a probabilistic model is the conceptual framework. This consideration more than any other affects the predicted range of outcomes.

Breadth of model. The greatest risk to the use of probabilistic analysis is the failure to include major, unrecognized drivers of uncertainty. Probabilistic analysis is frequently applied to reserves alone, without considering economic uncertainty in product prices, development schemes, and costs. If the objective is to understand the uncertainty in value, then the model should address those drivers of value and not just of reserves. This is an issue of "begin with the end in mind." The modeler must understand what uncertainty he is trying to quantify, i.e., the purpose and use of the analysis.

Given a clear understanding of the uncertainty to be quantified, the model builder should try to understand the potential drivers of that uncertainty. In part, this is an iterative process. First he builds the model and then tests it with preliminary inputs to determine the drivers. Finally the modeler can refine those inputs. But he must be careful from the beginning that he does not construct the model in the first place to exclude potentially critical features.

A comprehensive outline of what type of uncertainties may, and arguably should, be included can be found in reference 3. Use of a standard structure like the one described will assist the evaluator in the examination of the key uncertainties and is helpful for comparison between projects.

Paradigms or Model Structure. The greatest failure of probabilistic analysis occurs when the evaluator uses the wrong paradigm in the model.[6,7] Paradigm shifts, i.e., different concepts about the reality to be modeled, can cause wholesale differences in predicted ranges of outcomes between analysts and over time. Different technicians analyzing the same data can create different models and arrive at substantially different answers as a result of structuring the calculations differently. Reference 8 offers an example of this dynamic.

Different ideas and rules, correlations, sub-divisions of data, reservoir processes, etc., express themselves as differences in the structure of the model. For example, the inclusion of capital constraints on a development plan in a statistical play can significantly affect the results of the analysis. Alternatively, one analyst may decide that there is large uncertainty around the compressibility of the formation that another analyst may not recognize. One analysis may assume one drive mechanism while a subsequent analysis may recognize another.

Much discussion has been given to the participation of multi-disciplinary teams and peer review in the determination of the range of inputs to a model. These same techniques are equally applicable, and more valuable, at the stage of framing the types of uncertainties to include and the paradigms to investigate.

Calculation methods. There is often more difference between different types of calculations for the same results as there is among the results using one technique with different inputs. The range of calculated results for different techniques may not even overlap. Uncertainty analysis of decline curve projections can differ dramatically from a probabilistic treatment of a finite difference simulation model of the same field. The two are based on fundamentally different calculations with different inputs and different uncertainties associated with each. At a lower level, using different analytical waterflood techniques can result in meaningfully different ranges of outcomes.

Consequently, the evaluator should judiciously choose the calculation technique and may consider the calculation technique as one of the variables for examination when he audits his model.

Aggregation. Aggregation inside a probabilistic model can cause a falsely narrow distribution. It is well-established that aggregation of resultant distributions from various projects affects the range of the total distribution. Central limits theorem states that distributions of any shape, when summed, tend toward a normal distribution with progressively diminishing standard deviation. For multiplication of distributions, the results tend toward log normal with diminishing standard deviation.

The same principles of aggregation apply to the construction of a single probabilistic calculation. All else being equal, breaking down a calculation into more parts or more separate elements of uncertainty decreases the range of the final outcome.

A large-scale analysis may be broken down into too many parts. For instance, a practitioner may breakdown the calculation into too many separate areas of a field, separate stages of depletion, separate recovery mechanisms or other smaller parts. An uncorrelated well level analysis of decline curve uncertainty, when summed, will show less uncertainty than if the entire field decline curve is analyzed.

Excessive detail. Many have mistakenly assumed that a more comprehensive model is a more detailed one and thus a more accurate one. This is often not true. An analyst may opt to include too many separate contributors to uncertainty, causing the overall uncertainty of the major drivers to be masked by the uncertainty of the lesser drivers. In addition, it is possible to break the uncertainty into too many component parts. For example, one might model bulk rock volume uncertainty as the combination of four uncertainties related to seismic interpretation. Trying to add too many possible contributors to uncertainty in an effort to make a comprehensive model may lead to an excessively narrow result. A model should focus on that limited set of issues which dominate the resultant uncertainty.[9,10,11]

Another example illustrates both of these points. A cost estimate for a major project consisting of four concurrent parts may be described by the authors of the estimate as accurate to +/- 25%. A probabilistic analysis might incorrectly separate the estimate into the four parts and apply a range of +/-25% to each component of the estimate independently. Cost uncertainty at the total project level is then improperly reduced.

Length and thickness in the volumetric equation offers another good example. These two variables may not be independent, but the relationship is usually complex. There is no need for an anachronistic reduction of the uncertainty into these two inputs when they can be straightforwardly modeled as uncertainty in gross rock volume.

It is not improper to model at a fine scale or to include many uncertain elements. In fact, it may be most appropriate to separate different calculations and uncertainties in a probabilistic model. However, the implications of doing so may not be obvious or intended, especially when independence is assumed. The implications of building the model in this way should be tested, understood, and affirmed. If the implications do not reflect the intention of the modeler, then adjustments can be made in the structure of the calculations, the uncertainties included or the correlations among parts of the calculation.

Correlations. Correlations are a major driver of the final range of outcome. Particularly in cases where the model construction requires many distributions, attention must be paid to possible correlations.

Much has been written about how to model certain types of correlations. Unfortunately, correlations come in many forms and are often quite subjective and amorphous. Relationships are often not amenable to quantification and reduction into correlation coefficients as required by common software packages. Moreover, the correlation coefficients used in the common software packages address rank correlations, not value correlations. Alternative methods of implementing correlation can seem equally arbitrary.

Correlations between risk factors offer a good example. Risk factor distributions are usually binomial, yielding values of either zero or one. It is obvious that certain risks between prospects may be related, yet rank order correlation coefficients are awkward at best. Nonetheless, the dependence must be represented somehow in the calculations.

Correlations can sometimes be modeled best by combining distributions rather than using correlation algorithms.[12,13] For example, the effects of porosity and water saturation in a gas reservoir can be modeled together as hydrocarbon porosity. In this way, the analyst circumvents the need to quantify and quality control the correlation between the parameters.

Since overly narrow results are the unique danger of probabilistic analysis, the presumption of correlation is to be preferred. Suspected correlation should usually be included in the model and the effects of the correlation checked as described below.

Input distributions

Though less important than model architecture, input distributions affect the results of a probabilistic model. There are two technical considerations in the construction of each distribution to the probabilistic model: the functional form and the range. However, the consideration of biases and types of uncertainty form a foundation for the definition of the two technical parameters. Consequently, bias and uncertainty are the more important and are treated first below.

Before reviewing specific problems in the specification of form and range, it is important to understand what the probability density functions (pdf) are meant to express.

An input pdf is meant to express the evaluator's sense of the range of what the correct value might be. It is often not meant to capture a stochastic process since many, even most, of the inputs are not stochastic (random) in nature. Moreover, the evaluator does not and cannot know *a priori* what the relative probabilities are. A card game like black jack or poker can be modeled accurately because the rules and probabilities are known exactly. The petroleum evaluator does not have this privilege. The ranges of inputs to a simulation are subjective estimates of what the probabilities might be or of what the truth might be. In the end, the primary purpose of a distribution is to express the relative probability of a range of outcomes in the view of the evaluator.

Caldwell,[14] Capen,[15] Campbell,[16] and Bratvold[7] have described the types of psychological biases that can influence the judgment calls made by evaluators. These are the most meaningful considerations in the formation of input distributions as they can have order-of-magnitude impact on the results. Caldwell has written the most comprehensive treatment of the types of uncertainties generally encountered. Fylling[12] provides a useful examination of these uncertainties as it applies to petrophysical analyses.

Murtha[11] describes how distributions, i.e., range and form, can be constructed from three general sources: fundamental principles (primarily on form), expert opinions and historical or analog data. Selected issues related to form and range are outlined below.

However, the evaluator should bear in mind that form and range are a means and not the end of specifying inputs. The two elements of the input distributions that are propogated through a model directly to the final result are the mean and the standard deviation.[17] Consequently, the analyst should be more concerned with the mean and standard deviation of his inputs than with the functional form. The range is more important than the form since range generally has the greater impact on standard deviation.

Bias. Several good treatments have been given to the types and sources of inadvertent bias which can interfere with the objective quantification of uncertainty.[14,16,18,7,15] The evaluator should bear closely in mind the possibility of such bias in himself since the bias can have an order-of-magnitude level impact on the resultant range. Previously identified biases are briefly recapitulated below:

Confirmation bias. The tendency to "see what you believe."

Availability bias. The tendency to give undue weight to data which is readily available.

Recency or Primacy bias. The tendency to give undue weight to the most recent or the first data observed.

Vividness bias. The tendency to give undue bias to data which is particularly interesting or engaging.

Anchoring. The tendency not to adjust far enough away from some reference point.

Status quo bias. The tendency to quit examination and resist change.

Illusion of control bias. The tendency to overestimate the odds of success by failing to consider all of the parameters or drivers outside of the evaluator's control.

Motivational bias. The tendency to expect or endorse an answer which provides personal rewards.

Cognitive or Experience bias. Tendency to evaluate based upon the training and experience available to the evaluator without considering the limitations of its applicability in the current situation.

Overconfidence bias. As the single greatest source of faulty ranges, this topic deserves more treatment. Research conducted by Capen[15] and others illustrates that people tend to underestimate their range of uncertainty, even about topics with which they are familiar. Capen cites empirical research suggesting that one has trouble estimating ranges beyond the 70% confidence interval. It is easy to see how this might be the case since most of life is not concerned with accurately estimating very high or very low frequency events. Moreover, we don't encounter enough "trials" of low or high probability events to develop a good quantitative sense about them.

The uncertainty between the estimates of different people is generally larger than the uncertainty judged by a single person. As with model construction, formation of the range of uncertainty in an input distribution should represent the contributions of several analysts.

Ironically, Capen[15] and Bratvold[7] further observe, the perceived range of uncertainty is smaller when less is known about a subject. This is the "problem of one data point," the situation where psychological biases are most likely to come into play. With little data, one is likely to assume that the value is near the center of the distribution of possible outcomes. However, it could well be an extreme value. Without other data to provide a sense of scale to the uncertainty, people tend to underestimate the range of possible outcomes.

An analyst should be vigilant against these biases in the formation of the ranges. More importantly, systems can be created for the implementation of probabilistic analysis to reduce these biases.

Types of uncertainty. When an analyst sets about to specify an input distribution, he should create the distribution to reflect the major kinds of uncertainty present in the variable. A conceptual framework for the types of uncertainty was earlier addressed by Purvis[19] and Caldwell[14] and reviewed here.

Incomplete Data. The difficulty of assessing a range of results from a small or incomplete dataset. As described above, this is the area where the psychological biases are likely to impact the results.

Computational approximation. The uncertainty associated with different methods used to calculate an input parameter. In the same way that different calculation methods in the architecture of the model affect the results, so do different calculation methods for determining the uncertainty in inputs to the model. Though it is almost always overlooked, this uncertainty can be the largest uncertainty present.

For example, different correlations for fluid properties, different models for interpreting log responses,[12] and different velocity mapping techniques may all result in wholly different figures for the uncertain variables. If an analyst considered only one method, he might artificially reduce the actual uncertainty in the result.

Measurement uncertainty. The imprecise or inaccurate ability to measure a property. "Precision" relates to exactness while "accuracy" relates to how close the answer is to the truth. Using a postal meter to weigh a person might yield an answer precise to a fraction of an ounce, but the answer is not likely to be accurate. A probabilistic analysis is meant to capture the issue of accuracy, not of precision.

All measurements have some error rate associated with the accuracy of the results. If one assumes that the error is random, then the error may be assumed to be averaged to acceptably near zero. The real danger, and that which the analyst must be most aware of, is the possibility of systematic error.[20] Weighing the same person on 100 postal meters and averaging the results does nothing to remove the systematic error introduced by using an inaccurate technique.

Stochastic systems. Most of the inputs to a probabilistic analysis reflect uncertainty of estimation, but some inputs represent more truly stochastic (random) variables.

Of these types of uncertainties, the problems associated with incomplete data are the most insidious and potentially the most important. Computational approximation may be important and has often been ignored. Although hard to detect, systematic error of measurement has the potential for significant impact on the answer. The builder of the probabilistic model should be aware of which types of uncertainties he is dealing with.

Range. When the evaluator has taken due notice of potential biases and the types of uncertainties involved, he may proceed to the specification of range and form. The specifications follow from the particulars of the variable and situation.

The following discussion deals in more detail with the application of some of the ideas listed above.

Excessive extrapolation. Though the more common error is the underestimation of ranges, in some cases the range may be too broad. In the absence of constraining data, an evaluator's paradigm may exceed the limits of reality or the limits of reserves definitions. For example, in the situation of an evaluation with an unknown water contact, a reasonable distribution for the water contact would have an absolute upper limit at the lowest known hydrocarbon. Unless corrected by the circumspect analyst, the P10 of the resultant distribution would be located deeper than the lowest known hydrocarbon permitted by reserve definitions for the Proved category. At the other end of the distribution, the lower limit of the range of contacts would be limited only by the optimism of the engineer, expanding the median and P90 (i.e., Probable and Possible) estimates accordingly.

Scale. The scale of the data used and the scale to which it is applied can also affect the range of uncertainty. The most common scale difficulty is that of applying small scale data to large scale uncertainties. Understanding the differences between data used to form a distribution and the application of a distribution can be important.

The range of the average is not the same as the range of the population observed. The range of a population is an example of small scale data, but the average is applied large-scale over the entire reservoir. For example, the range of half-foot porosity values measured by logs in one well is sometimes used as the basis for average porosity throughout a new discovery. In doing so, the engineer assumes that the one well tested all the range of porosities present in the reservoir and in the same proportions as they are likely to occur on average. It is clear that unless the entire reservoir is just like the one well, the confidence interval of the average porosity measured in one well will not be the same as that for the entire reservoir.

The modeler must make some kind of assumption to get a reasonable range of values. Substituting the range measured in one well is one way to approximate the range. But it should be realized that if there are significant differences in lithology, the reservoir-scale average may not fall within the weight of probabilities.

Different analysis techniques. Though mentioned above, this idea remains poorly enough understood that it bears repeating. Actual values that are the basis for input distributions are often themselves the result of some sort of calculation. Bulk volume, for instance, can be the result of a seismic interpretation, or net pay can be the result of a log interpretation. The differences among calculation techniques can be greater than the differences in the calculation of multiple values by the same technique. For example, net pay may be consistent across several wells given a single set of net pay criteria. Changing the net pay criteria may yield a different range of uncertainty. Or, changing the interpretation algorithm may have the same effect. The final input distribution should acknowledge both the variability between wells and between calculation methods when both are judged to be significant. In this way, an analyst may best represent the range of uncertainty by making several diverse scenario-type calculations of the inputs and creating a distribution to reflect the range of results.

Use of Scenario Analysis. Keeping in mind that many variables are not stochastic in nature, the evaluator may find that certain situations are better handled by scenario analysis than probabilistic analysis.

In some cases there may be major discontinuities in the uncertainty of a particular parameter. For example, one might estimate only a 50% probability that a secondary recovery project will be implemented. Or, the price to be received for production from a remote field may center around two diverse estimates, depending on which pipelines are built at the time production begins.

In some situations of little data, the evaluator may not have sufficient data to determine a functional form or range, i.e., he may not be able to determine a range with associated relative probabilities, or the distribution may be highly subjective. In this case also, scenario analysis may be the better option. As always, the scenarios should cover a wide range of possibilities.

In these cases the results of a probabilistic analysis may be clearer, less subjective and more useful if certain uncertainties are not modeled and the assumptions clearly stated. Instead, two cases can be run, one with each assumption, as a scenario analysis.

Form. The form of a distribution does impact the resultant distribution, though to a lesser degree than the range, unless the uncertainty is driven by one major factor. In some cases there may be a theoretical basis for the selection of a particular distribution. However, as stated before, the ultimate objective of the form is to reflect the relative probabilities as judged by the evaluator.

Normal and log normal distributions are quite common in nature and often cited as the theoretically appropriate form. For example, the distribution of permeability values in a reservoir is often assumed to be log normal while porosity is observed to be normally distributed. It has also been observed that the population of field sizes appears to be log normal.

In such a case, the form of the distribution can be used to define part of the range as well. For example, a log normal distribution is fully defined by a mean and a P90 value. Given the form and a single value, the entire distribution is defined.

It should be noted, however, that some of the theory and empirical evidence supporting normal and log normal distributions relates to populations of data. It is a fallacy to suppose that the uncertainty about an individual in a population necessarily follows the same form as the distribution of actual outcomes of the population as a whole. Once a member of the population is identified, e.g., a prospect is identified on seismic, the uncertainty is specific to that member only and may take any form which reflects the uncertainty as judged by the evaluator. Similarly, it is a fallacy to suppose that the uncertainty in the average of a population is necessarily the same form and range as the distribution of the population itself.

For example, if someone were to try to estimate the population of New York City, he might observe that the distribution of city sizes in the United States is log normally distributed. He could then take a guess at the population, assign a probability level and end up with a range of possible outcomes with a high side in excess of the actual value.

That is not to say that information of a different scale or information for an entire population is not useful for defining distributions. These factors should not be regarded as the touchstone for the creation of inputs.

When considering alternative forms, the evaluator should be conscious of the skewness in the form. Skewed forms lead to more difference between separate measures of central tendency and generally to greater standard deviation.[21]

Testing the Model

It has been said, "Start right. Work right. Finish right." Care in the initial construction and execution of the probabilistic model is not sufficient either to obtain reliable results or to obtain the valuable insights sought. The model builder must interrogate, audit and test the model.[11,22]

Plainly, one should examine the architecture of the model. Running the model under different paradigms (assumptions), with different rules, with and without correlations, and even using different calculation methods may offer insights and refinements to the model. One may also hold different parts of the analysis constant and examine the interactions with other parameters and the impact on the resultant distribution.

Of course, the input distributions should be subjected to sense-checks and sensitivities. Though range and form were carefully selected, the input distributions may not reflect the evaluator's uncertainty. This testing may occur after running the model since the results of the model often give the evaluator more data about the implications of the inputs. Following are some suggestions of how assumptions may be tested and thus refined.

- Restate probabilities as odds. This can be done with risk probabilities and with continuous variables. For example, the odds are 4 to 1 that a value is less than a P80 figure.
- Compare probabilities or odds of the input distributions to those for more familiar types of events, such as the likelihood of drawing two pair in a game of five card stud. (5%)[23]
- Examine the probability of drawing values between multiple pairs of values, e.g., there is a 50% chance of average porosity being between 18% and 22%.
- Examine the distributions implied by intermediate results or smaller parts of the simulation. For example, examine the product of porosity and gas saturation or the total number of successful prospects.
- Examine the distribution of combinations of inputs and the results for individual realizations.
- Examine pairs of values or other results created by correlations.

As a result of this kind of audit and testing, the final version of the model is likely not to be the same as the initial version. The changes should be regarded as refinements and improvements to accurately reflect the uncertainty as understood by the evaluator.

Perspectives on Results

Good construction and good testing should result in a model worthy of use in decision-making. However, we should note the limits of even a good model. The result of a good probabilistic analysis can be described as an accurate reflection of the uncertainty in the final outcome as judged by the evaluator and as a result of the uncertainties considered in the calculation. Though it yields more information, probabilistic analysis can be as subjective as deterministic analysis.[24]

More importantly, the extremes of a resultant distribution are more poorly defined and are subject to greater error than central estimates. (See also ref. 6.) Though we have faith in the central estimates, it would be ridiculous to conclude that a value just outside the simulated range was absolutely impossible. Said another way, there is more uncertainty in the location of the P10 value than there is in the value of the mean due to both model construction and convergence.

While the mean of a resultant distribution converges quickly (i.e., with fewer trials), many more trials are required to converge on a stable value of a probability level nearer the extremes. In all but the most computationally intensive models like those involving large finite difference simulations, this limitation can be overcome simply by letting the computer run longer. In large models or when using experimental design or other approximations,[13] this consideration may affect the stability of the P10 or P90 answer.

The extremes of the distribution are more likely to change with changes in the model architecture, input ranges and even pdf forms. Changes with minor or negligible effects in the mean result can meaningfully change P10 or P90. Jordan offers an example of this phenomenon.[25] Ironically, these extreme values are the ones used to determine reserves.

It is easy for an evaluator to place excess faith in a model of reality that is not correct or not consistent with reserve criteria. It seems to be the nature of engineers, and humans in general, to be inherently optimistic about future developments and the adequacy of available data to describe reality. This can result in the inclusion of too much in a model, or distributions that are too broad or too narrow. An evaluator should recognize the limits of available data and, in some cases, depending on the use of the evaluation, the limits imposed by reserve criteria. Recognizing these limits may not change the way the modeling is conducted but should certainly affect the way the results are viewed.

Best Practice and Heuristics

Many of the pitfalls and shortcomings of probabilistic analyses can be avoided by a few simple steps.[1,4,7,26,27,28,29] Among the most important are:

- Users and builders of probabilistic analyses should be well-grounded in probability and statistics.
- Careful attention should be paid to the design ("framing") and testing of the architecture of the model.
- Models should be built with contributions and review by a group of people.
- The model should be subjected to sense-checks and sensitivity analysis before being finalized.
- When a type of situation is encountered repeatedly in a company (e.g., prospect analysis), the company should maintain a single model for use in every case and ensure that the model is implemented consistently.
- Companies should maintain databases of actual results so that improvements can be made to the model, to the training of evaluators on probabilities, and to provision of a database of analogous data from which to conceive input distributions for future projects.

To this list of guidelines, I offer the following heuristics:

- The purpose of probabilistic analysis is to understand risks and uncertainty. Underestimation of the range of results represents a failure in the analysis. Consequently, the presumption of correlation should usually be favored.
- More detailed models are not necessarily better models. The issue of aggregation within a model can lead to excessively narrow distributions of results. The model should focus on the most important uncertainties.
- The greatest uncertainty for an input to a model or within a model can be the method of calculation. For example, differences between methods can be much larger than differences for one method with variable inputs. Input distributions may sometimes be defined by determining values using multiple methods.
- Correlations are often poorly defined and awkwardly programmed. If not properly sense-checked after the fact, the results may not be consistent with the judgment of the evaluator. In many cases, correlations can be circumvented by the combination of variables.
- In forming the inputs to the model, the evaluator should consciously be aware of his own biases and bear in mind the types of uncertainty at hand.
- The purpose of an input distribution is to reflect the uncertainty as determined by the evaluator.
- It is a fallacy to suppose that the uncertainty about an individual in a population necessarily follows the same form as the distribution of actual outcomes of the population as a whole. Similarly, it is a fallacy to suppose that the uncertainty in the average of a population is necessarily the same form and range as the distribution of the population itself.
- A model should be thoroughly audited, tested and refined before it is deemed useful.

These ideas will go a long way toward making judicious and consistent evaluations of risk.

Conclusions

The above observations about the process of building a probabilistic model lead to several conclusions:

1. The architecture of a probabilistic model has more impact on the results than the range or form of the inputs. The architecture should be carefully designed and tested. In this way, the evaluator gains the main benefits of probabilistic analysis.
2. Though it yields more information, probabilistic analysis is not without subjectivity. The result of a good

probabilistic analysis can be described as an accurate reflection of the uncertainty in the final outcome as a result of the considered uncertainties in a calculation as judged by the evaluator.
3. The extremes of a resultant distribution are more poorly defined and sensitive to change than central estimates. There is more uncertainty in the location of the P10 value than there is in the value of the mean due to both model construction and, potentially, convergence. Poor model-building causes excessively narrow distribution of results and thus leads to higher estimates of "reasonably certain" values.
4. The possibility of a quantum shift in uncertainty is completely undefined by distributions of results, making it the greatest danger of probabilistic analysis. In some cases, scenario analysis is aptly used together with or instead of probabilistic analysis to examine wholly different paradigms.

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