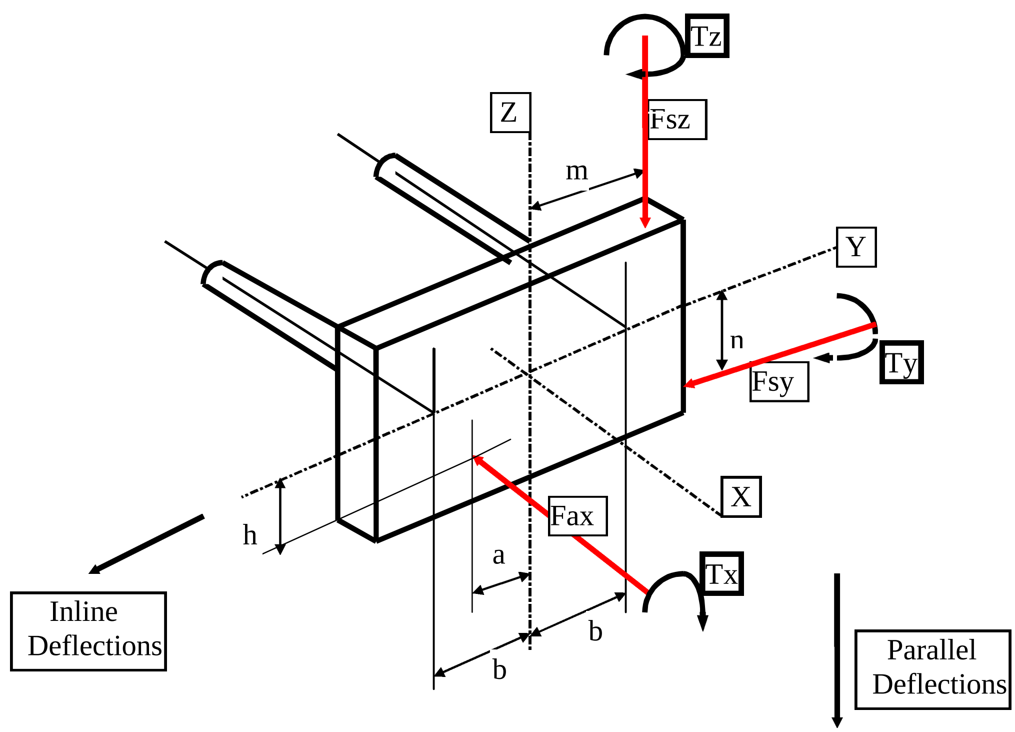
ISO / TR 16806:2003

Published 2003–02–15

PNEUMATIC FLUID POWER - CYLINDERS
LOAD CAPACITY OF PNEUMATIC SLIDES
AND THEIR PRESENTATION METHOD

This Technical Report (TR) describes an analytical technique for rating the load capacity of pneumatic slides – how much load and torque can be placed on the tool plate, and at what distance. Developments of loading equations are given in Annex A of the TR, which is shown in these succeeding pages.

The tool plate is attached to a pair of guide rods that support the loads, as shown in the following figure:

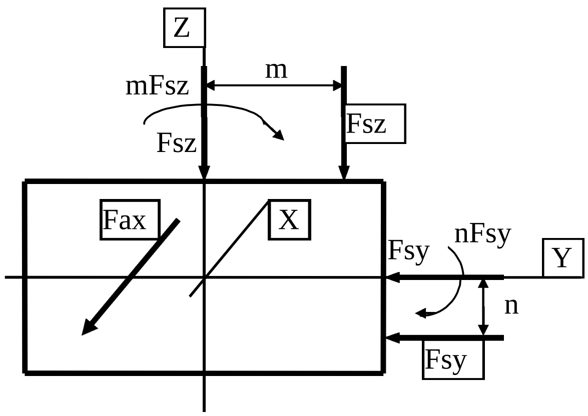


<u>ANNEX A (INFORMATIVE)</u>

DEVELOPMENT OF RATING EQUATIONS

Consider a tool plate with all possible force and torque loads applied as shown in Fig. 1 of the standard. These loads must be resolved into components that act on the rod guide bearings in order to determine if they are within the bearing capacities.

Begin with the side load forces Fsy and Fsz at their arbitrary locations and resolve these into equivalent side loads, and moments, on the principle axes (see Fig. 1A).



Fsy is translated to axis Y with corresponding moment nFsy acting clockwise around the X axis.

Fsz is translated to axis Z with corresponding moment mFsz acting clockwise around the X axis.

Fig. 1A

Now consider how all of the forces and moments act on a free body of the tool plate and its guide rods. The following three sketches describe the resulting reaction loads on the bearings:

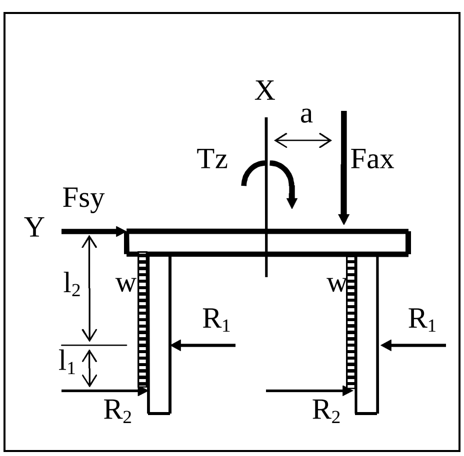


Fig.2A

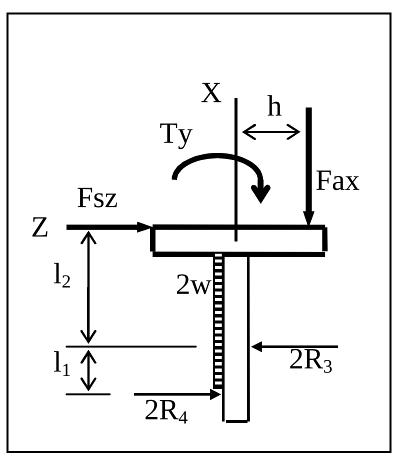


Fig. 3A

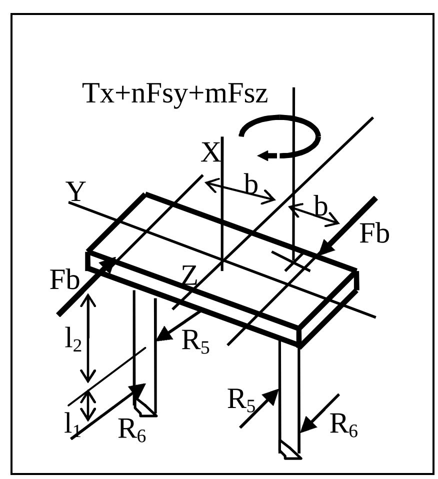


Fig. 4A

From the assumptions stated in the Scope of the standard (clause 1), forces in the X axis will be absorbed by the piston rod. Therefore, these will not be considered in the balance of forces on the tool plate and guide rods. However, there must be a balance of forces in the Y and Z axes of each of the three sketches, plus a balance of moments from all three axes.

Consider sketches 2A and 3A and the balance of moments about the axis in the plane of the sketch (Note: weight of the guide rods will only be effective if they are subject to gravity):

In Fig. 2A, $\sum Mz = 0$

Consider moments about a line passing through the reactions R_2, at the intersection of the X axis:

$$2R_1(l_1) = Fsy(l_1+l_2) + Fax(a) + Tz + 2w(l_1+l_2)[(l_1+l_2)/2]$$

$$R_1 = \left[Fsy(l_1+l_2) + Fax(a) + Tz + w(l_1+l_2)^2\right] / 2l_1$$

Consider moments about a line passing through the reactions R_1, at the intersection of the X axis:

$$2R_2(l_1) + 2wl_1(l_1/2) = Fsy(l_2) + Fax(a) + Tz + 2wl_2(l_2/2)$$

$$R_2 = \left[Fsy(l_2) + Fax(a) + Tz + w(l_2^2 - l_1^2)\right] / 2l_1$$

In Fig. 3A, $\sum My = 0$

Consider moments about a line passing through the reactions R_4, at the intersection of the X axis:

$$2R_3(l_1) = Fsz(l_1+l_2) + Fax(h) + Ty + 2w(l_1+l_2)[(l_1+l_2)/2]$$

$$R_3 = \left[Fsz(l_1+l_2) + Fax(h) + Ty + w(l_1+l_2)^2\right] / 2l_1$$

Consider moments about a line passing through the reactions R_3, at the intersection of the X axis:

$$2R_4(l_1) + 2wl_1(l_1/2) = Fsz(l_2) + Fax(h) + Ty + 2wl_2(l_2/2)$$

$$R_4 = \left[Fsz(l_2) + Fax(h) + Ty + w(l_2^2 - l_1^2)\right] / 2l_1$$

Now, consider Fig. 4A and resolve the moments in the plane perpendicular to the X axis, into a couple Fb(2b) also acting in a plane perpendicular to the X axis. Then:

$$Fb(2b) = Tx + nFsy + mFsz$$

Twist of the tool plate is prevented by equal forces from each guide rod acting to oppose the loads Fb. Each guide rod, then, is subject to the load Fb plus reactions from the rod bearings as shown in Fig. 4A. A balance of moments on each guide rod, perpendicular to the Y axis, can then be made:

Consider moments about a point on the guide rod at the intersection of the reaction R_6:

$$R_5(l_1) = Fb(l_1+l_2)$$

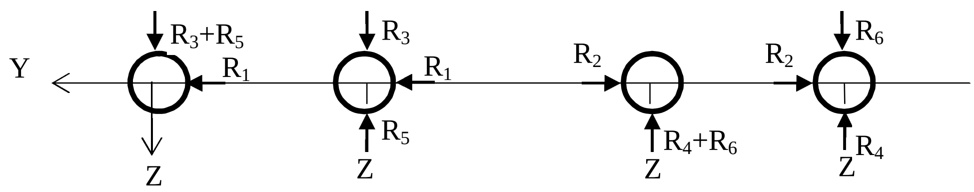
$$R_5 = (l_1+l_2)\left[Tx + nFsy + mFsz\right] / 2bl_1$$

Consider moments about a point on the guide rod at the intersection of the reaction R_5:

$$R_6(l_1) = Fb(l_2)$$

$$R_6 = (l_2)\left[Tx + nFsy + mFsz\right] / 2bl_1$$

All of the reaction loads on the guide rod bearings are now determined. However, they are applied in different directions as shown in Fig. 5A below, for each bearing:



Bearings near tool plate   Bearings farthest from tool plate

Fig. 5A

The total load on each bearing is the vector sum of the several reactions as follows:

$$R_A^2 = R_1^2 + (R_3 \pm R_5)^2 \qquad R_B^2 = R_2^2 + (R_4 \pm R_6)^2$$

were R_A is the total reaction on each bearing near the tool plate (use + sign for one bearing; - sign for the other bearing), and R_B is the total reaction on each bearing farthest from the tool plate. If a slide design consists of only one bearing on each guide rod, then l_1 is the length of that bearing, and only the reaction R_A will exist.

From a summation of forces on each guide rod in figures 2A, 3A, 4A, it can be observed that forces R_1, R_3, R_5 are larger than any of the other forces acting on a rod. Therefore, the bearing closest to the tool plate has the maximum load at all times. Furthermore, the bearing where forces R_3 and R_5 are additive, is the highest loaded bearing and can be used for the capacity rating.

Then: $(R_A)^2_{max} = R_1^2 + (R_3 + R_5)^2$

Substituting values into the equation for $(R_A)^2_{Max}$ yields:

$$(R_A)^2_{Max} = \{[Fsy(l_1+l_2) + w(l_1+l_2)^2 + Fax(a) + Tz] / 2l_1\}^2 + \{[Fsz(l_1+l_2) + w(l_1+l_2)^2 + Fax(h) + Ty] / 2l_1 + (l_1+l_2)[Tx + nFsy + mFsz] / 2bl_1\}^2$$

Multiplying both sides by the scaling factor, f, and substituting the following terms:

$F = f\,(R_A)_{Max}$ $\quad A = 2l_1/f$ $\quad B = 2l_1 / f\,(l_1+l_2)$ $\quad W = w(l_1+l_2)$

the following is obtained:

$$F^2 = \{(Fsy + W)/B + [(a)Fax+Tz]/A\}^2 + \{[Fsz\,(1+m/b)+W]/B + [(h)Fax+Ty]/A + [nFsy+Tx]/bB\}^2$$

This is the equation described in clause 5.4.1 of the standard.

DEFLECTION EQUATIONS

Consider two possible directions of deflections as described in Fig. 1 of the standard, where the loads causing each type of deflection are described in figures 2A and 3A. The deflection is different for these two cases because the system of connected guide rods resist the bending in different ways. But, for either of these cases a guide rod can be represented by an equivalent beam loaded as shown in Fig. 6A:

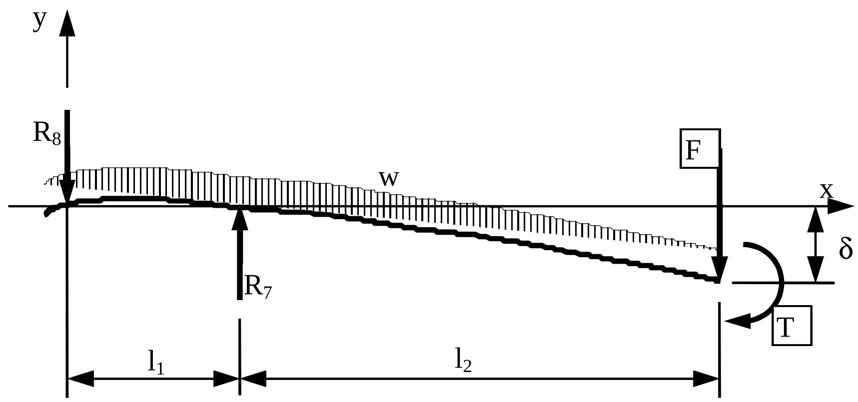


Fig. 6A

Here all of the loads and torque are represented by the symbols F and T .

Taking successive balances of moments around R_7 and R_8, the reactions are determined as follows:

$$R_7 = [F(l_1 + l_2) + T + \cancel{2}w(l_1 + l_2)\frac{(l_1 + l_2)}{\cancel{2}}]/l_1 \qquad R_8 = [Fl_2 + T + \cancel{2}wl_2(\frac{l_2}{\cancel{2}}) - \cancel{2}wl_1(\frac{l_1}{\cancel{2}})]/l_1$$

The double integration method is now used to determine the equation of the deflected beam:

For $0 \le x \le l_1$

$$d^2y/dx^2 = M/EI = \frac{-R_8 x - \cancel{2}wx(x/\cancel{2})}{EI}$$

$$dy/dx = -\frac{wx^3}{3EI} - \frac{R_8 x^2}{2EI} + C_1$$

$$y = -\frac{wx^4}{12\,EI} - \frac{R_8 x^3}{6EI} + C_1 x + C_2$$

At $x = 0$; $y = 0$ and $C_2 = 0$

At $x = l_1$; $y = 0$ and $C_1 = \dfrac{wl_1^3}{12\,EI} + \dfrac{R_8 l_1^2}{6EI}$

For $l_1 \le x \le (l_1 + l_2)$

$$d^2y/dx^2 = M/EI = \frac{-R_8 x + R_7(x - l_1) - \cancel{2}wx(x/\cancel{2})}{EI}$$

$$dy/dx = \frac{-wx^3}{3EI} + \frac{(R_7 - R_8)x^2}{2EI} - \frac{R_7 l_1 x}{EI} + C_3$$

$$y = \frac{-wx^4}{12\,EI} + \frac{(R_7 - R_8)x^3}{6EI} - \frac{R_7 l_1 x^2}{2EI} + C_3 x + C_2$$

At $x = l_1$; dy/dx for $x \le l_1$ = dy/dx for $x \ge l_1$

and $C_3 = \dfrac{wl_1^3}{12E} + \dfrac{R_8 l_1^2}{6EI} + \dfrac{R_7 l_1^2}{2EI}$

At $x = l_1$; $y = 0$ and $C_4 = -\dfrac{R_7 l_1^3}{6EI}$

Now, the desired result is an expression for maximum deflection. Therefore, use the equation where:

$$l_1 \le x \le (l_1 + l_2)$$

Which is:

$$y = -\frac{w}{12EI}x^4 + \frac{(R_7 - R_8)}{6EI}x^3 - \frac{R_7 l_1}{2EI}x^2 + \frac{2l_1^2(3R_7 + R_8) + wl_1^3}{12EI}x - \frac{R_7 l_1^3}{6EI}$$

Substituting $y = -\delta$ at $x = (l_1 + l_2)$ into the above equation yields:

$$-\delta = \frac{R_7 l_2^3}{6EI} - \frac{R_8 l_2(2l_1 + l_2)(l_1 + l_2)}{6EI} + \frac{w(l_1 + l_2)}{12EI}\left[l_1^3 - (l_1 + l_2)^3\right]$$

Now, substituting the expressions for R_7 and R_8 into this yields:

$$-\delta = \frac{l_2^3}{6EIl_1}\left[F(l_1 + l_2) + T + w(l_1 + l_2)^2\right] - \frac{l_2(2l_1 + l_2)(l_1 + l_2)}{6EIl_1}\left[Fl_2 + T + w(l_2^2 - l_1^2)\right] + \frac{w(l_1 + l_2)}{12EI}\left[l_1^3 - (l_1 + l_2)^3\right]$$

Combining terms and substituting $W = w(l_1 + l_2)$ yields:

$$-\delta = \frac{-F\left[2l_2^2(l_1 + l_2)\right]}{6EI} - \frac{Tl_2(2l_1 + 3l_2)}{6EI} - \frac{Wl_2}{12EI}(3l_2^2 + l_1 l_2 - l_1^2)$$

Which can be simplified to:

$$d = \frac{[4F(C) + 2T + W(D)]}{12EI \big/ l_2(2l_1 + 3l_2)} \qquad \text{where:} \qquad C = \frac{l_2(l_1 + l_2)}{(2l_1 + 3l_2)}$$

$$D = \frac{3l_2^2 + l_1 l_2 - l_1^2}{(2l_1 + 3l_2)}$$

Now the values for "F" and "T" will depend upon the orientation of the slide - either Inline or Parallel as shown in Fig. 1 of the standard.

Inline Deflections (also see Fig. 2A)	Parallel Deflections (also see Fig. 3A)
F = Fsy and T = (a)Fax + Tz	F = Fsz and T = (h)Fax + Ty
δ = [4Fsy(C) + 2(a)Fax + 2Tz + W(D)] / H	δ = [4Fsz(C) + 2(h)Fax + 2Ty + W(D)] / H

These are the equations described in clause 5.4.2 of the standard, where:

$$H = \frac{12EI}{l_2(2l_1 + 3l_2)}$$ as described in clause 4.3 of the standard.