ISO 8778: 2003 – Annex B

Published 2003-03-15

Development of equations for relative humidity, density and error analysis

B.1 Statement of the problem

When compressed air in a container or conduit is equated to its atmospheric equivalent, the water vapour content is often ignored in the calculation process. However, the change of state will affect the density at atmospheric conditions. Although the density change may not affect an equivalent-state calculation (pressure and temperature), the water content could be important in those calculations where the change affects system calculations, such as a dehumidifying process.

The following analysis develops equations for determining the relative humidity and density at atmospheric conditions for a mixture of air and water vapour at pressurized conditions.

B.2 Symbols and constants

The following symbols and constants are used in the following equations.

NOTE All pressures and temperatures are absolute.

M_a	=	28,967 g/mole (molecular weight of dry air)
M_{wv}	=	18,016 g/mole (molecular weight of water vapour)
m_a	=	mass of dry air
m_{wv}	=	mass of water vapour
p_o	=	pressure of the atmosphere (mixture of air and water vapour)
p_1	=	pressure of mixture of air and water vapour in the compressed state
p_{ao}	=	partial pressure of the dry air at atmospheric conditions
p_{a1}	=	partial pressure of the dry air in the compressed state
p_{wv1}	=	partial pressure of the water vapour in the compressed state
p_{wvo}	=	partial pressure of the water vapour at atmospheric conditions, if it contained all of the water vapour from the compressed state
p_{s0}	=	saturation pressure of the water vapour at atmospheric temperature
p_{s1}	=	saturation pressure of the water vapour at the temperature of the compressed state
R_a	=	0,28700 $Nm^4/g°K$ (gas constant for dry air)
R_{wv}	=	0,46145 $Nm^4/g°K$ (gas constant for water vapour)
T_o	=	temperature of the atmosphere
T_1	=	temperature of the compressed state
ϕ_o	=	relative humidity of the air at atmospheric conditions if it contained all of the water vapour from the compressed state
ϕ_1	=	relative humidity of the air in the compressed state
ϕ'_o	=	arbitrarily specified relative humidity of the air at atmospheric conditions
ρ_0	=	density of the mixture at atmospheric conditions with relative humidity equivalent to the compressed state
ρ'_0	=	density of the mixture at atmospheric conditions with arbitrary relative humidity

B.3 Relative humidity

In general, the pressure of an air/water vapour mixture is the sum of its two partial pressures:

$$p_{mixture} = p_{dry\ air} + p_{water\ vapour}$$

Using the symbols and constants given above, the following can be stated for both compressed and atmospheric conditions:

$$p_{a1} = p_1 - p_{wv1} \quad \text{and} \quad p_{ao} = p_o - p_{wvo} \qquad (1)$$

From the definition of relative humidity (at compressed and at atmospheric conditions):

$$\phi_1 = p_{wv1} / p_{s1} \quad \text{and} \quad \phi_o = p_{wvo} / p_{so}$$

The partial pressures of the water vapour then become:

$$p_{wv1} = \phi_1 p_{s1} \quad \text{and} \quad p_{wvo} = \phi_o p_{so} \qquad (2)$$

If it is assumed that none of the water vapour will condense, the specific humidity will be the same at compressed and atmospheric conditions. Then, the following will hold:

$$\frac{p_{wv1} M_{wv}}{p_{a1} M_a} = \frac{p_{wvo} M_{wv}}{p_{a0} M_a}$$

Substituting equations (1) and (2) into this and solving for ϕ_o yields:

$$\phi_o = \phi_1 (p_o / p_1)(p_{s1} / p_{so}) \qquad (3)$$

If the assumption of no condensation is not true, the above result will be greater than 100 %. This is an indication that the calculation is not valid. In this case, the relative humidity will only be 100 %.

B.4 Density

The density of mixed air at atmospheric conditions is composed of two parts, and an equation of state can be written for each part as follows:

$$p_{ao} V = m_a R_a T_o \quad \text{and} \quad p_{wvo} V = m_{wv} R_{wv} T_o$$

The mass of the mixture will be equal to the sum of the mass of each component:
and

$$m = m_a + m_{wv} = \frac{p_{a0} V}{R_a T_0} + \frac{p_{wvo} V}{R_{wv} T_0}$$

$$\frac{m}{V} = r_0 = \frac{p_{a0}}{R_a T_0} + \frac{p_{wvo}}{R_{wv} T_0}$$

Substituting the partial pressure expressions from equations (1) and (2) into this yields:

$$r_0 = \frac{p_0}{R_a T_0} - \frac{f_0 p_{s0}}{T_0}\left(\frac{1}{R_a} - \frac{1}{R_{wv}}\right) \tag{4}$$

Finally, substituting the expression for relative humidity in the compressed state from equation (3):

$$r_0 = \frac{p_0}{R_a T_0} - f_1\left(\frac{p_0}{p_1}\right)\frac{p_{s1}}{T_0}\left(\frac{1}{R_a} - \frac{1}{R_{wv}}\right) \tag{5}$$

B.5 Error analysis

If equation (5) gives the density of a mixture at atmospheric conditions, what is the error if an arbitrary value for relative humidity at atmospheric conditions is assumed instead?
The error to be evaluated is defined as follows:

$$\% Error = \frac{deviation}{correct \quad value}(100) = \frac{r_0 - r'_0}{r_0}(100) = \left[1 - \frac{r'_0}{r_0}\right](100)$$

From equation (4), the density of a mixture for an arbitrary relative humidity at atmospheric conditions will be:

$$r'_0 = \frac{p_0}{R_a T_0} - f'_0\left(\frac{p_{s0}}{T_0}\right)\left(\frac{1}{R_a} - \frac{1}{R_{wv}}\right)$$

Substituting this and equation (5) into the error formula yields:

$$\% \ Error = \left[1 - \frac{1 - f'_0\left(\frac{p_{s0}}{p_0}\right)\left(1 - \frac{R_a}{R_{wv}}\right)}{1 - f_1\left(\frac{p_{s1}}{p_1}\right)\left(1 - \frac{R_a}{R_{wv}}\right)}\right](100) \tag{6}$$

A series of sample density error calculations is shown in Table B.1, using two cases of arbitrary relative humidity at atmospheric conditions $\phi'_o = 65$ % and $\phi'_o = 0$ %. It is assumed the atmospheric pressure is equal to 760 mm Hg.

Table B.1 – Sample density error calculations

Assumed pressurized state conditions					Calculated at atmospheric pressure of 760 mm Hg and temperature of 20 °C			
p_1 mm H_g abs	**p_1** mbar, gauge	**T_1** °C	**f_1** % RH	**p_{s1}** mm H_g abs	**p_{s0}** mm H_g abs	**f_0** % RH	**% Error** at 65 % RH	**% Error** at 0 % RH
8500	10 280	20	100	17,5	17,5	8,94	0,488	-0,078
5500	6 300	20	100	17,5	17,5	13,82	0,446	-0,120
2300	2 050	20	100	17,5	17,5	33,04	0,279	-0,288
1000	320	20	100	17,5	17,5	76,00	-0,096	-0,666
8500	10 280	40	100	55,1	17,5	28,15	0,322	-0,246
5500	6 300	40	100	55,1	17,5	43,51	0,188	-0,380
2300	2 050	40	100	55,1	17,5	104,04	-0,343	-0,914
1000	320	40	100	55,1	17,5	239,29	-1,550	-2,127
8500	10 280	60	100	149	17,5	76,13	-0,098	-0,667
5500	6 300	60	100	149	17,5	117,65	-0,463	-1,035
2300	2 050	60	100	149	17,5	281,34	-1,931	-2,511
1000	320	60	100	149	17,5	647,09	-5,370	-5,970
8500	10 280	20	75	17,5	17,5	6,71	0,508	-0,058
5500	6 300	20	75	17,5	17,5	10,36	0,476	-0,090
2300	2 050	20	75	17,5	17,5	24,78	0,351	-0,216
1000	320	20	75	17,5	17,5	57,00	0,070	-0,499
8500	10 280	40	75	55,1	17,5	21,11	0,383	-0,184
5500	6 300	40	75	55,1	17,5	32,63	0,283	-0,285
2300	2 050	40	75	55,1	17,5	78,03	-0,114	-0,684
1000	320	40	75	55,1	17,5	179,47	-1,012	-1,587
8500	10 280	60	75	149	17,5	57,10	0,069	-0,500
5500	6 300	60	75	149	17,5	88,24	-0,204	-0,774
2300	2 050	60	75	149	17,5	211,01	-1,295	-1,871
1000	320	60	75	149	17,5	485,31	-3,820	-4,411
8500	10 280	20	50	17,5	17,5	4,47	0,527	-0,039
5500	6 300	20	50	17,5	17,5	6,91	0,506	-0,060
2300	2 050	20	50	17,5	17,5	16,52	0,423	-0,144
1000	320	20	50	17,5	17,5	38,00	0,236	-0,332
8500	10 280	40	50	55,1	17,5	14,08	0,444	-0,123
5500	6 300	40	50	55,1	17,5	21,75	0,377	-0,190
2300	2 050	40	50	55,1	17,5	52,02	0,114	-0,455
1000	320	40	50	55,1	17,5	119,65	-0,481	-1,053
8500	10 280	60	50	149	17,5	38,06	0,235	-0,332
5500	6 300	60	50	149	17,5	58,83	0,054	-0,515
2300	2 050	60	50	149	17,5	140,67	-0,667	-1,240
1000	320	60	50	149	17,5	323,54	-2,316	-2,898
8500	10 280	20	25	17,5	17,5	2,24	0,547	-0,019
5500	6 300	20	25	17,5	17,5	3,45	0,536	-0,030
2300	2 050	20	25	17,5	17,5	8,26	0,494	-0,072
1000	320	20	25	17,5	17,5	19,00	0,401	-0,166
8500	10 280	40	25	55,1	17,5	7,04	0,505	-0,061
5500	6 300	40	25	55,1	17,5	10,88	0,472	-0,095
2300	2 050	40	25	55,1	17,5	26,01	0,340	-0,227
1000	320	40	25	55,1	17,5	59,82	0,045	-0,524
8500	10 280	60	25	149	17,5	19,03	0,401	-0,166
5500	6 300	60	25	149	17,5	29,41	0,311	-0,257
2300	2 050	60	25	149	17,5	70,34	-0,047	-0,616
1000	320	60	25	149	17,5	161,77	-0,854	-1,428
8500	10 280	20	10	17,5	17,5	0,89	0,558	-0,008
5500	6 300	20	10	17,5	17,5	1,38	0,554	-0,012
2300	2,05	20	10	17,5	17,5	3,30	0,537	-0,029
1000	320	20	10	17,5	17,5	7,60	0,500	-0,066
8500	10 280	40	10	55,1	17,5	2,82	0,541	-0,025
5500	6 300	40	10	55,1	17,5	4,35	0,528	-0,038
2300	2 050	40	10	55,1	17,5	10,40	0,476	-0,091
1000	320	40	10	55,1	17,5	23,93	0,358	-0,209
8500	10 280	60	10	149	17,5	7,61	0,500	-0,066
5500	6 300	60	10	149	17,5	11,77	0,464	-0,103
2300	2 050	60	10	149	17,5	28,13	0,322	-0,246
1000	320	60	10	149	17,5	64,71	0,003	-0,567

B.6 Observations

This analysis describes the error resulting when air in a compressed state is equated to atmospheric conditions, and an arbitrary value is assumed for its relative humidity. The last two columns of Table B.1 describe the error caused by this effect, at the two sets of conditions defined in this International Standard.

Each group of four rows in Table B.1 shows a different temperature or relative humidity in the compressed state. The pressure varies from high to low in each group of four rows.

The shaded cells in Table B.1 highlight conditions in which the relative humidity at atmospheric conditions has exceeded 100 %, which would result in condensation. This demonstrates limitations to the concept that water vapour in air at the compressed state may be equated to the atmospheric state.

A positive error indicates that the arbitrary change results in a lower density. A negative error yields a higher density.

B.7 Conclusion

In general, condensation will occur when compressed air with a high relative humidity and high temperature is expanded to atmospheric conditions. The equations are then limited in their applicability in these conditions.

For equivalent state calculations, where air is not actually expanding, the density error is minor for the two cases of arbitrary relative humidity. It is cautioned, however, that these are only sample calculations and may differ from actual applications.