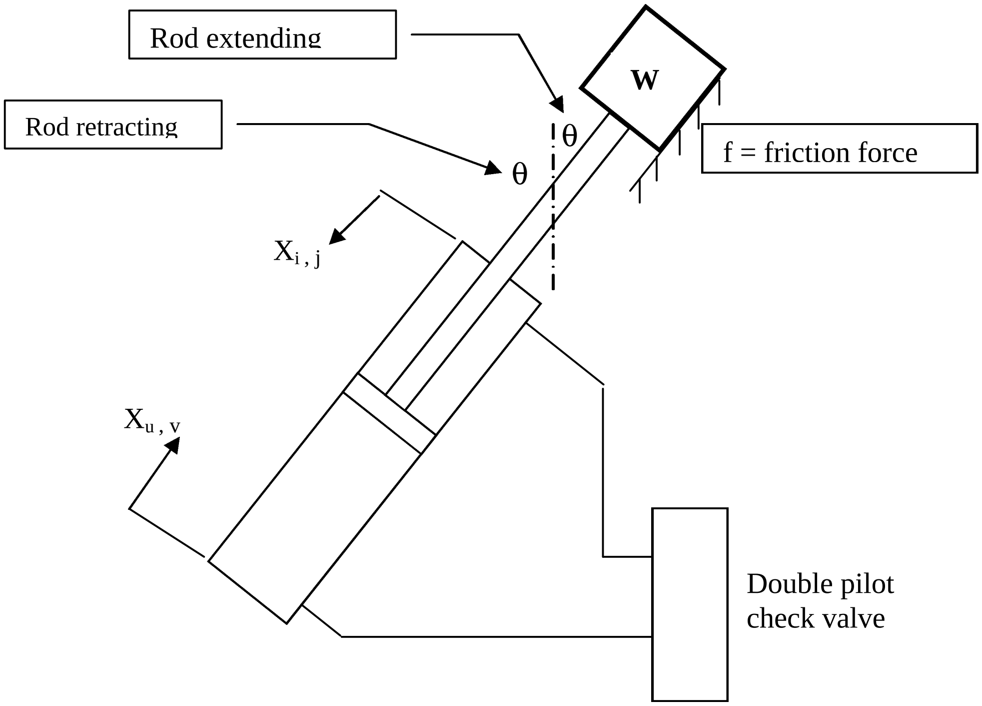
Double Piloted Operated Check Valve

Cylinder Deceleration Analysis

28 June 2002

Given: A cylinder system connected to a double pilot operated check valve as shown:



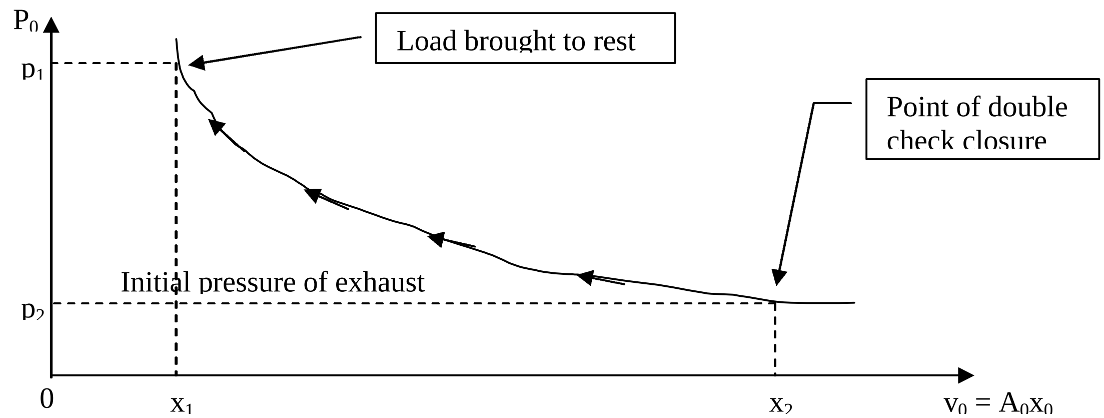
When the double pilot check valve closes, air is trapped on both sides of the cylinder. If the cylinder was in motion with an attached load (W), the trapped air on the side opposing motion will be compressed; and the trapped air on the driving side will expand.

Problem: determine the maximum pressure of the compressed air, the minimum pressure of the expanded air, and the position of the piston when motion first stops. It is understood that subsequent bouncing of the load may occur.

Assumptions and definitions:

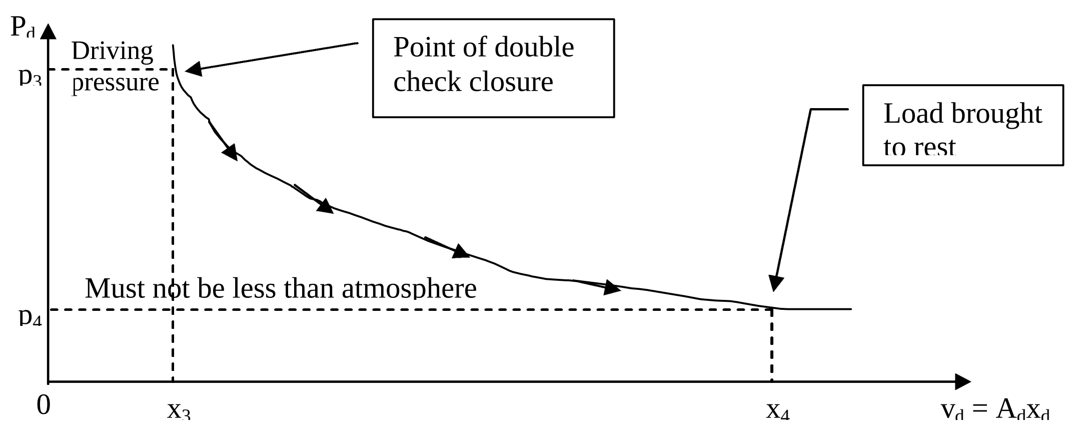
1. There is no air leakage into or out of the system after the double pilot check valve closes.
2. The compressed and expanded air will only be in the cylinder (discount the connecting lines).
3. The process can begin at any position of the cylinder stroke. The piston position is defined by a variable x, which is zero at each end of the cylinder.
4. The angle θ is measured between the upward vertical (anti-gravity), and the direction of motion.
   Examples: rod pointing up, retracting and rod pointing down extending: θ = 180°;
   rod pointing up, extending and rod pointing down, retracting: θ = 0°;
   horizontal, extending or retracting: θ = 90°.
5. Friction is constant during the process.

Analysis of the trapped air:
Consider a PV diagram of the compressed air:



Note that $v_0 = A_0x_0$, where: A_0 = cylinder area opposing motion.
x_0 = distance measured to the piston from the end of the cylinder being approached. At $x_0 = 0$, the piston will have contacted the cylinder end.

Consider a PV diagram for the expanded air on the opposite side of the piston:



Note that $v_d = A_dx_d$, where: A_d = cylinder area driving the motion.
x_d = distance measured to the piston from the end of the cylinder where motion started. At $x_d = 0$, the piston leaves the cylinder end to begin travel.

Note, also, that p_4 must be ≥ atmospheric pressure, otherwise the check opens in the double pilot operated check valve and atmospheric air is sucked in. This would violate assumption 1 and the PV diagram would not be valid.

All pressures are absolute.

Pressure relationships in the PV diagram will follow the perfect gas laws for an adiabatic process. Adiabatic is chosen because the process is rapid and it will be assumed that heat losses are negligible for one deceleration action.

Then: pv^k = constant and,

$$p_o v_o^k = p_1 v_1^k = p_2 v_2^k$$

$$p_o = p_2\left(\frac{v_2}{v_o}\right)^k = p_2\left(\frac{x_2}{x_o}\right)^k$$

also: $$p_1 = p_2\left(\frac{x_2}{x_1}\right)^k$$

$$p_d v_d^k = p_3 v_3^k = p_4 v_4^k$$

$$p_d = p_3\left(\frac{v_3}{v_d}\right)^k = p_3\left(\frac{x_3}{x_d}\right)^k$$

also: $$p_4 = p_3\left(\frac{x_3}{x_4}\right)^k$$

Energy Balance:

In order to decelerate the load:

Work done by the Cylinder = Change in Kinetic Energy + Change in Potential Energy

Work = [Differential force on the piston – friction] • [distance traveled]

$$= \int_{x_i}^{x_j} (p_d A_d - p_o A_o - f)dx = \int_{x_i}^{x_j}\left[p_3\left(\frac{x_3}{x_d}\right)^k A_d - p_2\left(\frac{x_2}{x_o}\right)^k A_o - f\right]dx$$

Note: Work is positive in the direction of motion.

$$= p_3 x_3^k A_d \int_{x_3}^{x_4}\frac{dx}{x^k} - p_2 x_2^k A_o \int_{x_1}^{x_2}\frac{dx}{x^k} - f\int_{x_3}^{x_4} dx$$

$$= \frac{p_3 x_3^k A_d}{-k+1}\left(x_4^{-k+1} - x_3^{-k+1}\right) - \frac{p_2 x_2^k A_o}{-k+1}\left(x_2^{-k+1} - x_1^{-k+1}\right) - f\left(x_4 - x_3\right)$$

Note: valid only if $k \neq 1$. Thus, only adiabatic is valid; not isothermal.

$$= \frac{p_3 x_3 A_d}{1-k}\left[\left(\frac{x_3}{x_4}\right)^{k-1} - 1\right] - \frac{p_2 x_2 A_o}{1-k}\left[1 - \left(\frac{x_2}{x_1}\right)^{k-1}\right] - f\left(x_4 - x_3\right)$$

Work $$= \frac{p_3 x_3 A_d}{k-1}\left[1 - \left(\frac{x_3}{x_4}\right)^{k-1}\right] + \frac{p_2 x_2 A_o}{k-1}\left[1 - \left(\frac{x_2}{x_1}\right)^{k-1}\right] - f\left(x_4 - x_3\right)$$

Change in Kinetic Energy = $\frac{W}{2g}\left(V_2^2 - V_1^2\right) = \frac{W}{2g}\left(0 - V_{MAX}^2\right) = -\frac{WV_{MAX}^2}{2g}$

Where: V_{MAX} = velocity of load at the time of the double check valve closing.

Change in Potential Energy = $W\left(x_4 - x_3\right)\cos\theta$

Then:

$$\frac{p_3 x_3 A_d}{k-1}\left[1-\left(\frac{x_3}{x_4}\right)^{k-1}\right]+\frac{p_2 x_2 A_o}{k-1}\left[1-\left(\frac{x_2}{x_1}\right)^{k-1}\right]-f\left(x_4-x_3\right)=-\frac{WV_{MAX}^2}{2g}+W\left(x_4-x_3\right)\cos\theta$$

Now, the cylinder stroke, $s = x_2 + x_3 = x_1 + x_4$

Let η = the portion of stroke s where the double pilot check valve closes.

Then: $\eta s = x_3$ and $(1 - \eta s) = x_2$

Then the expression:

$$\frac{x_2}{x_1}=\frac{x_2}{x_2+x_3-x_4}=\frac{1}{1+\frac{x_3}{x_2}-\frac{x_4}{x_2}}=\frac{1}{1+\frac{\eta s}{(1-\eta)s}-\frac{x_4}{(1-\eta)s}}=\frac{1-\eta}{1-\frac{x_4}{s}}$$

and the final equation becomes:

$$\boxed{\frac{p_3 \eta s A_d}{k-1}\left[1-\left(\frac{\eta s}{x_4}\right)^{k-1}\right]+\frac{p_2(1-\eta)sA_o}{k-1}\left[1-\left(\frac{1-\eta}{1-\frac{x_4}{s}}\right)^{k-1}\right]+\frac{WV_{MAX}^2}{2g}-\left(W\cos\theta+f\right)\left(x_4-\eta s\right)=0}$$

For a given application, where the 10 physical parameters are given (bore, stroke, etc.), the only unknown in the above equation is x_4.

With a solution for x_4, the following are found:

$$p_1 = p_2\left(\frac{x_2}{x_1}\right)^k = p_2\left[\frac{(1-\eta)}{1-\frac{x_4}{s}}\right]^k \quad \text{and} \quad p_4 = p_3\left(\frac{x_3}{x_4}\right)^k = p_3\left[\frac{\eta s}{x_4}\right]^k$$

It is important in certain cases to determine the compression ratio. If the ratio is above 10:1, ignition of any oil in the air may occur, causing an explosion. Therefore:

Compression ratio = $r = \frac{x_2}{x_1} = \frac{1-\eta}{1-\frac{x_4}{s}}$

Solving for x_4 :

The solution for x_4 in the above, boxed equation is found by using an Excel worksheet. The terms are subdivided as follows:

Terms	English units	Metric units
$A = \frac{p_3 hsA_d}{k-1}\left[1-\left(\frac{hs}{x_4}\right)^{k-1}\right]$	$\left(\frac{\#}{in.^2}abs.\right)in.in.^2 = in.\#$	$(bar \cdot abs)\left(\frac{10^5\,Pa}{1bar}\right)\left(\frac{1\frac{N}{m^2}}{1Pa}\right)mm \cdot mm^2\left(\frac{1m^3}{1000^3 mm^3}\right) = \frac{Nm}{10^4}$
$B = \frac{p_2(1-h)sA_o}{k-1}\left[1-\left(\frac{1-h}{1-\frac{x_4}{s}}\right)^{k-1}\right]$	$\left(\frac{\#}{in.^2}abs.\right)in.in.^2 = in.\#$	$(bar \cdot abs)\left(\frac{10^5\,Pa}{1bar}\right)\left(\frac{1\frac{N}{m^2}}{1Pa}\right)mm \cdot mm^2\left(\frac{1m^3}{1000^3 mm^3}\right) = \frac{Nm}{10^4}$
$C = \frac{WV_{MAX}^2}{2g}$	$\frac{\#\frac{ft.^2}{\sec^2}\left(12\frac{1n.}{ft.}\right)}{2\left(32.2\frac{ft.}{\sec^2}\right)} = \frac{in.\#}{5.367}$	$\frac{kg\left(\frac{m^2}{\sec^2}\right)\left(9.81\frac{N}{kg}\right)}{9.81\frac{m}{\sec^2}} = Nm$
$D = -(W\cos q + f)(x_4 - hs)$	$\#\ in.$	$\left[kg\left(\frac{9.81N}{kg}\right) + N\right](mm)\left(\frac{1m}{1000mm}\right) = \left(\frac{9.81+N}{1000}\right)Nm$

Thus: A+B+C+D = 0

The Excel spreadsheet is organized in columns, starting with the unknown x_4 ; then columns of A, B, C, D, and a column of SUM. An entry is made for the first trial value of x_4 at ηs, and succeeding rows will have small increments. Each cell in the A, B, C, D columns will contain a calculation using the trial value of x_4 , and the SUM column will add the 4 columns together. When the SUM column is zero, the trial value of x_4 for that entry is the solution to the boxed equation.

Columns of p_1 and p_4 can be included for each row to automatically determine those values for each trial; as well as the compression ratio, r.

Input required:

- p_2 = initial pressure of exhaust
- p_3 = driving pressure
- A_o = area opposed to motion
- A_d = driving area
- f = friction force
- s = stroke
- η = portion of stroke when PO check closes.
- W = load attached, plus piston & rod weight
- V_{MAX} = velocity when P.O. check closes
- θ = orientation angle relative to gravity.