Two-Car Collision at City Intersection

by Frank Owen, Alpha Omega Engineering, Inc. (www.aoengr.com), all rights reserved
August 2012

This example is taken from the book Technische Analyse von Verkehrsunfällen anhand von Beispielen, but the analysis is my own and somewhat different from the analysis in the book.

Events

Figure 1 – Intersection where collision occurred (Google Maps, August 2012)

Figure 1 shows the intersection in Ettlinger, Baden Württemberg, Germany, where this traffic accident occurred. The two vehicles involved were a Nissan Vanette (vehicle A, m_A = 1335 kg) and a Toyota Corolla (vehicle B, m_B = 1005 kg). The Nissan (A) was stopped at a stop sign on Friedrichstraße and pulled out to turn left onto Pforzheimerstraße. When it was most of the way across the intersection, it was struck in the rear, left corner by the Toyota (B), which was proceeding right on Pforzheimerstraße. See the diagram in Figure 2 to understand the collision scenario.

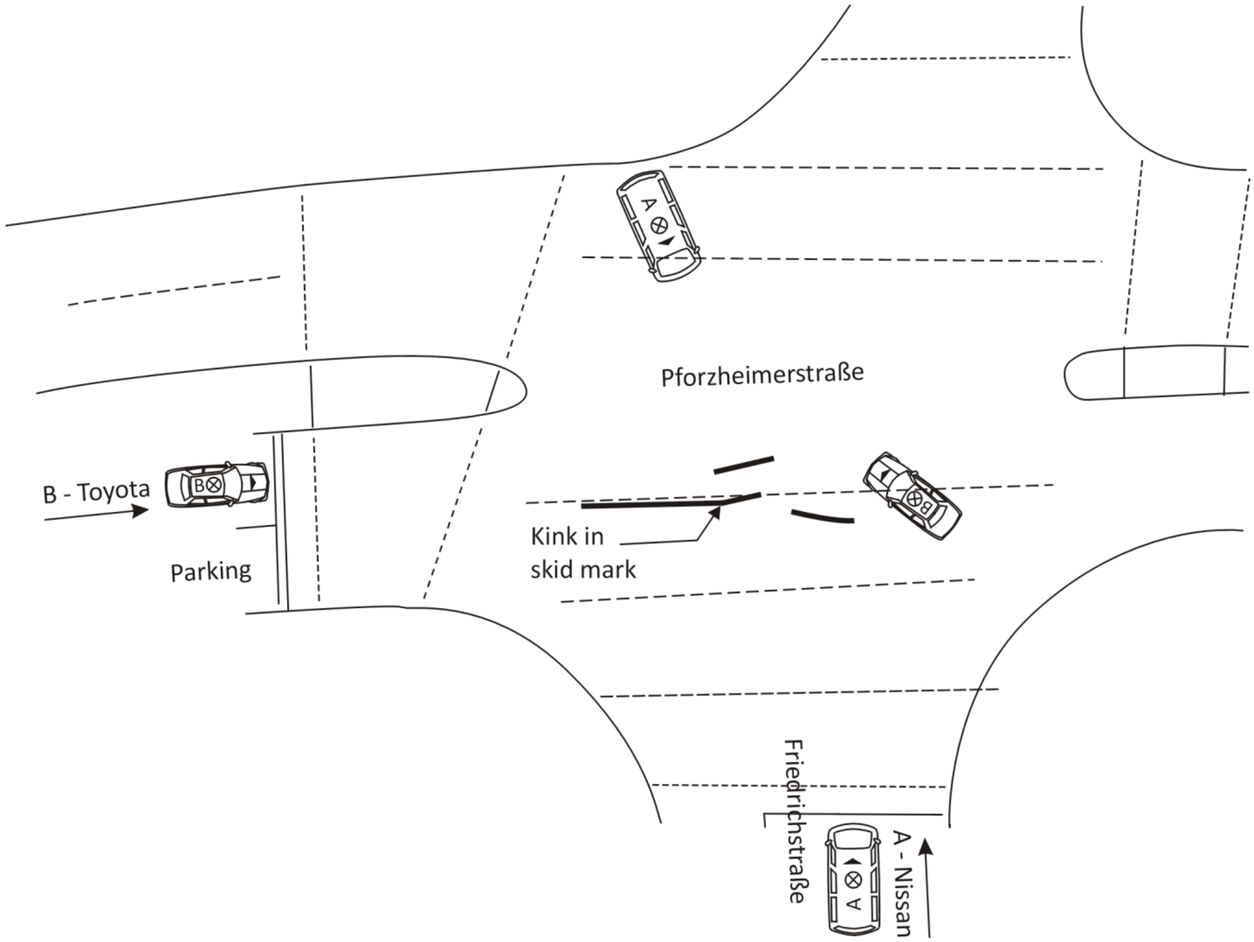

Figure 2 – Diagram of accident scene

The collision occurred on 14 September 1996 at 9:10 PM, so after dark. The roadway surface was dry asphalt. The drawing shows also the skid marks noted by accident investigators. Noteworthy are pre-collision skid marks made by the Toyota (B). They exhibit a kink, a sharp change in direction toward the end of the skid marks. The left, corner of the Toyota was substantially damaged. The left front wheel was pushed back significantly and could not roll after the collision. The other three wheels were functional. The left rear corner of the Nissan (A) was lightly damaged, yet all four wheels were still rollable after the collision. The driver of the Nissan (A) was not wearing his seatbelt, and he struck his head on the windshield of the vehicle.

The driver of the Toyota (B) was given a breathalyzer test at the accident scene and was found to have a blood-alcohol content of 0.12%, above the legal limit. The posted speed limit for Pforzheimerstraße is 50 kph. The Toyota (B) was damaged to the extent that the front right tire became locked in the collision.

Questions to be answered by a reconstruction of this accident are

1) What was the speed of the Toyota (B) prior to braking? Was it excessive? Was the accident caused by an excessive speed that may have caused the driver of the Nissan (A) to proceed from a stop because he assumed he could clear the intersection before the arrival of the Toyota (B)?
2) Or if the speed of the Toyota (B) was not excessive, was the accident the fault of the Nissan (A) driver because he either did not see the Toyota (B) or he underestimated the time it would take him to clear the intersection?

Analysis

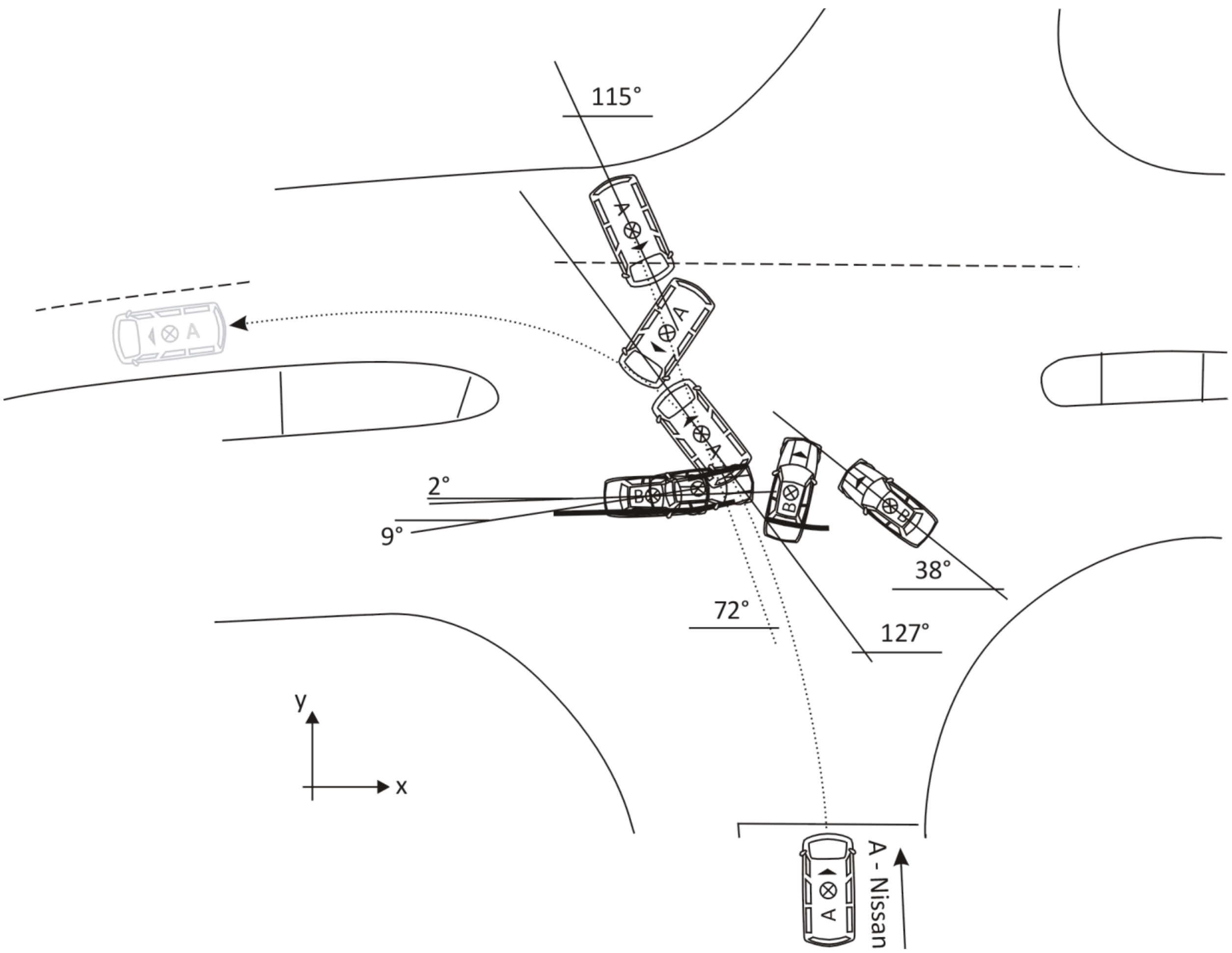

Figure 3 – Assumed post-collision spin-out

Figure 3 shows an analysis of the collision and the subsequent spin-out of the two vehicles. The skid marks of the Toyota (B), particularly the kink in the skid marks, identify the point of impact of the two vehicles. Shown also is the probable intended trajectory of the Nissan (A) as it left the stop sign on Friedrichstraße to make the left turn onto Pforzheimerstraße. It is not clear from the drawings contained in the German source whether or not Pforzheimerstraße is two-lane or four-lane at this point, but analyzing the Google Maps photo (Figure 1) of today, it looks as if the street is two-lane. The position of the Nissan (A) at the point of impact is not obvious due to the lack of skid marks made by this vehicle. The trajectory of this vehicle shown in Figure 3 is a spline-line smoothly connecting the start position of the Nissan with the intended end position of the vehicle, had the collision not occurred. The Nissan (A) has been placed and oriented at the point of impact so that its left, rear corner is in contact with the Toyota (B), and it is oriented tangent to its assumed trajectory.

The post-collision movement of the Nissan (A) is also unknown, due to the absence of skid marks from this vehicle. The vehicle obviously rotated through 168°. This rotation was produced by the impulse delivered by the Toyota (B) striking the Nissan (A) on the corner with a blow that is not directed in any way through or near its center of gravity. Such a blow tends to produce rotation and not translation. The movement in the *y* direction of the Nissan (A) after the collision was thus not the result in any large degree of an impulse from the Toyota (B) but rather due to its pre-collision *y* velocity. As can be seen from the skid marks of the Toyota (B), during the collision its front, left corner got entangled with the rear, left corner of the Nissan (A) to an extent that the Nissan (A) actually dragged it a bit in the *y* direction to cause the kink in the skid mark (a 7°-deflection from the pre-crash velocity of the Toyota (B)).

But the post-collision trajectory of the Nissan (A) is still unknown. The impulse from the Toyota (B) produced rotation, primarily, but it would also deflect the mass center of the Nissan (A) a bit to the right. As can be seen in Figure 3, the end position of the Nissan (A) is a bit to the right of where it would be had simply continued along its tangent line at the point of impact and rotated into its final configuration. This is the result of the right-ward impulse delivered to the Nissan (A) by the Toyota (B).

(The question naturally arises, why are there skid marks from the Toyota (B) but not from the Nissan (A)? This could be caused by simply by tires of different manufacture. Also, a vehicle can slide on a roadway without leaving skid marks. Notice that the skid marks left by the Toyota (B) prior to the crash were left during maximum braking. In this state the vehicle has pitched forward, adding more load to its front tires. This extra force would also tend to leave skid marks.)

Calculations

The Nissan (A) turned through a total of 168°, starting with a heading angle relative to the post-collision velocity of 19°. The average sine for this angle range was 0.648. The Toyota (B) turned through an angle of 146° starting at an angle of 0° between the post-collision heading and velocity. The average sine for this range is 0.704. If we use $\mu = 0.8$ for dry asphalt, and note that all wheels of the Nissan (A) rolled after the accident and one wheel of the Toyota (B) was locked after the accident, then the drag factors—f_A and f_B—are, respectively,

$$f_A = 0.648 \cdot \mu = 0.518$$

$$f_B = (0.704 \cdot 0.75 + 0.25) \cdot \mu = 0.622$$

The post-collision skid distances were respectively 8.56 m and 9.42 m. Thus the post-collision velocities are

$$v_{Af} = \sqrt{2 \cdot f_A \cdot g \cdot d_{Af}} = \sqrt{2 \cdot 0.518 \cdot 9.81 \frac{m}{sec} \cdot 8.56\, m} = 9.33 \frac{m}{sec} = 33.6\, kph \text{ @ } 108°$$

$$v_{Bf} = \sqrt{2 \cdot f_B \cdot g \cdot d_{Bf}} = \sqrt{2 \cdot 0.622 \cdot 9.81 \frac{\text{m}}{\text{sec}} \cdot 9.42\ \text{m}} = 10.73 \frac{\text{m}}{\text{sec}} = 38.6\ \text{kph @ } 7°$$

The pre-collision velocity directions are known or assumed. So we can use impulse/momentum to calculate the pre-collision velocities.

$$m_A \cdot v_{Aix} + m_B \cdot v_{Bix} = m_A \cdot v_{Afx} + m_B \cdot v_{Bfx}$$

$$m_A \cdot v_{Aiy} + m_B \cdot v_{Biy} = m_A \cdot v_{Afy} + m_B \cdot v_{Bfy}$$

$$\tan(\theta_{Ai}) = \frac{v_{Aiy}}{v_{Aix}}$$

$$\tan(\theta_{Bi}) = \frac{v_{Biy}}{v_{Bix}}$$

The unknowns are v_{Aix}, v_{Aiy}, v_{Bix}, and v_{Biy}. Thus these four unknowns can be found. Recouching this system in matrix format

$$\begin{bmatrix} m_A & 0 & m_B & 0 \\ 0 & m_A & 0 & m_B \\ \tan(\theta_{Ai}) & -1 & 0 & 0 \\ 0 & 0 & \tan(\theta_{Bi}) & -1 \end{bmatrix} \cdot \begin{bmatrix} v_{Aix} \\ v_{Aiy} \\ v_{Bix} \\ v_{Biy} \end{bmatrix} = \begin{bmatrix} m_A \cdot v_{Afx} + m_B \cdot v_{Bfx} \\ m_A \cdot v_{Afy} + m_B \cdot v_{Bfy} \\ 0 \\ 0 \end{bmatrix}$$

This can be set up in Excel for solution (see attached spreadsheet). The result is

$$\begin{bmatrix} v_{Aix} \\ v_{Aiy} \\ v_{Bix} \\ v_{Biy} \end{bmatrix} = \begin{bmatrix} v_{Aix} \\ v_{Aiy} \\ v_{Bix} \\ v_{Biy} \end{bmatrix} \text{kph} = \begin{bmatrix} -25.6 \\ 34.0 \\ 58.5 \\ 2.04 \end{bmatrix} \text{kph}$$

So

$$v_{Ai} = 42.5\ \text{kph @ } 127°$$

$$v_{Bi} = 58.6\ \text{kph @ } 2°$$

These are the velocities immediately prior to the collision. The velocity of the Toyota (B) prior to braking is even higher than this. The pre-crash skid marks of the Toyota (B) attest to a deceleration prior to the crash. The skid marks of the Toyota (B) before the crash are It is also known that full braking as indicated by the skid marks is not suddenly applied but rather is preceded by a gradual build-up in braking force over a period of about 1.0-1.5 seconds (Singh). To arrive at the steady-state speed prior to any reaction, we use the relation for full braking

$$\int_{x1}^{x2} a_{12} dx = \int_{v1}^{v2} dv$$

where 1 signifies the point at the start of skidding and 2 signifies the point of impact ($v_2 = v_{Bi}$). The deceleration during this phase is

$$a_{12} = -g \cdot f_{B12} = -9.81\ \mathrm{m/\,sec^2} \cdot 0.8 = -7.8\ \mathrm{m/sec^2}$$

So

$$v_1 = v_2 - a_{12} \cdot (x_2 - x_1) = 58.6\ \mathrm{kph} + 7.8 \frac{\mathrm{m}}{\mathrm{sec^2}} \cdot \frac{1}{(5.57\ \mathrm{m})} \cdot 3.6 \frac{\mathrm{kph}}{\mathrm{m/sec}} = 63.6\ \mathrm{kph}$$

Now we use a reaction time of 1.25 sec to increase the braking from nothing to full braking. During this phase we shall assume an average braking force of ½ the full braking force. This amounts to a linear increase in braking during this phase.

$$v_0 = v_1 - a_{01} \cdot t_{01} = 63.6\ kph + \frac{7.8}{2} \frac{\mathrm{m}}{\mathrm{sec^2}} \cdot 1.25\ \mathrm{sec} \cdot 3.6 \frac{\mathrm{kph}}{\frac{\mathrm{m}}{\mathrm{sec}}} = 81.3\ \mathrm{kph}$$

This is the calculated speed of the Toyota (B) prior to any braking reaction.

Conclusions

The fault for this collision lies with the driver of the Toyota (B). His blood alcohol level exceeded the maximum limit. Also he was exceeding the posted speed of 50 kph by a wide margin. This was after dark in a municipality. Had he been going at the posted speed, the Nissan (A) would have cleared the intersection well before the Toyota (B) arrived at the collision point.

References

Engineering Consultancy Neikes, Dillenburg, in Technische Analyse von Verkehrsunfällen anhand von Beispielen – Band I, GTÜ (Gesellschaft für Technische Überwachung mbH), 2000 Edition, Stuttgart.

Singh, Jay, “A Simple Closed Form Solution for the Displacement Constrained Problem of Braking to a Terminus Velocity“, Accident Reconstruction Journal, Volume 22, No. 4, Waldorf, MD, July/August 2012.

Ettlinger Unfall

Drawing analysis

Scale of drawing:

Width of Pforzheimerstr. At crosswalk:	17.13	m	8	cm - measured on screen with ruler
Toyota skidmark before kink	5.57	m	2.6	cm - measured on screen with ruler
dAf	8.56	m	4	cm - measured on screen with ruler
dBf	9.42	m	4.4	cm - measured on screen with ruler

Speed calculations:

mA	1335	kg
mB	1005	kg
ThetaAi	127	deg
ThetaAf	295	deg
delThetaA	168	deg
ThetaBi	2	deg
ThetaBf	144	deg
delThetaB	146	deg

Angle A			sin(A)	Angle B		sin(B)
	19	187	0.325568	0	146	0
	24		0.406737	5		0.087156
	29		0.48481	10		0.173648
	34		0.559193	15		0.258819
	39		0.62932	20		0.34202
	44		0.694658	25		0.422618
	49		0.75471	30		0.5
	54		0.809017	35		0.573576
	59		0.857167	40		0.642788
	64		0.898794	45		0.707107
	69		0.93358	50		0.766044
	74		0.961262	55		0.819152
	79		0.981627	60		0.866025
	84		0.994522	65		0.906308

89	0.999848	70	0.939693
94	0.997564	75	0.965926
99	0.987688	80	0.984808
104	0.970296	85	0.996195
109	0.945519	90	1
114	0.913545	95	0.996195
119	0.87462	100	0.984808
124	0.829038	105	0.965926
129	0.777146	110	0.939693
134	0.71934	115	0.906308
139	0.656059	120	0.866025
144	0.587785	125	0.819152
149	0.515038	130	0.766044
154	0.438371	135	0.707107
159	0.358368	140	0.642788
164	0.275637	145	0.573576
169	0.190809		0.703983
174	0.104528		
179	0.017452		
184	0.069756		
189	0.156434		
	0.64788		

mu	0.8						
Drag factors							
fA	0.518						
fB	0.622						
g	9.810	m/sec^2					
vAf	9.331	m/sec	33.59268	kph	@	108	°
vBf	10.725	m/sec	38.60824	kph	@	7	°
vAfx	-10.381	kph					
vAfy	31.949	kph					
vBfx	38.320	kph					

vBfy 4.705 kph

Solution for pre-crash velocities:

$$\begin{vmatrix} 1335 & 0 & 1005 & 0 \\ 0 & 1335 & 0 & 1005 \\ -1.327 & -1 & 0 & 0 \\ 0 & 0 & 0.034921 & -1 \end{vmatrix} \cdot \begin{vmatrix} vAix \\ vAiy \\ vBix \\ vBiy \end{vmatrix} = \begin{vmatrix} 24653.81 \\ 47379.98 \\ 0 \\ 0 \end{vmatrix} \begin{matrix} \text{kg*kph} \\ \text{kg*kph} \\ \\ \\ \end{matrix}$$

Inverse:

$$\begin{vmatrix} vAix \\ vAiy \\ vBix \\ vBiy \end{vmatrix} = \begin{vmatrix} 1.92E-05 & -0.00055 & -0.73423 & -0.55274 \\ -2.55E-05 & 0.00073 & -0.02564 & 0.733507 \\ 0.00097 & 0.000731 & 0.975324 & 0.734233 \\ 3.39E-05 & 2.55E-05 & 0.034059 & -0.97436 \end{vmatrix} \cdot \begin{vmatrix} 24653.81 \\ 47379.98 \\ 0 \\ 0 \end{vmatrix} \begin{matrix} \text{kg*kph} \\ \text{kg*kph} \\ \\ \\ \end{matrix}$$

Product:

$$\begin{vmatrix} vAix \\ vAiy \\ vBix \\ vBiy \end{vmatrix} = \begin{vmatrix} -25.58488 \\ 33.95229 \\ 58.51705 \\ 2.04346 \end{vmatrix} \text{ kph}$$

vAi	42.5	kph
vBi	58.6	kph

Check:

ThetaAi	127.0	°
ThetaBi	2	°

Skidding acceleration	7.8	m/sec^2

Pre-skid vB	63.63	kph
t01 (reaction time)	1.25	sec
Pre-reaction vB	81.29	kph